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ON THE LOCAL MAP OF MANIFOLDS

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Let us consider a n-dimensional differentiable manifold M of class C^{∞}. Let g be identical mapping of M. To every point $x \in M$ let there correspond just one pair (f_x, U_x), where f_x is a transformation of the class C^{∞} of M onto itself defined on a neighborhood U_x of a point $x \in M$ so, that $f_x(x) = x$ and that for an arbitrary curve c going through the point $x \in M$, the curves $f_x(c)$ and $g(c)$ have an analytic contact of the first order, but no contact of the second order at the point $x \in M$. We can now write briefly

$$f_x \in \dot{j}^1_x g \cap (\dot{j}^1_x(g) - \dot{j}^2_x(g)) = \mathcal{J}^1_x g$$

when $\dot{j}^r_x g$ is an infinitesimal jet of the order r (r-jet). We shall speak briefly about a combined manifold M_f.

It is straightforward that a correspondence between two projective, affine, ... spaces and a tangent homology, afinity, ... of this correspondence is a special case of a notion introduced above. It is possible to show that we can associate so called linearizating tensor (introduced by E. Čech), which plays a fundamental role in the theory of correspondences between two projective, affine, ... spaces, globally with a combined manifold M_f.

Further a relation among linear connections on M and the linearizating tensor associated with the combined mani-

-fold M_f will be shown. It appears that two connections which are in certain correspondence defined by the relation $M \to M_f$ have common torsion tensors.

1. Let M be an n-dimensional differentiable manifold of the class C^∞. Let us denote by $T_a(M)$ the tangent vector space of M at a point $a \in M$ and let $T(M)$ be the tangent bundle space over M. Let $P(M, GL(n,R))$ be a principal fibre bundle over a base manifold M with Lie structural group $GL(n,R)$ and with a projection p of P on M. We say that the vector $\tau \in T_b(P)$ is vertical if it is tangent to the fibre going through the point $b \in P$. For each $g \in G = GL(n,R)$ we denote by Dg the right translation of the manifold P corresponding to the element g.

Let $\mathcal{L}$ be the Lie algebra of the group G. We denote by σ a differentiable representation of G on a vector space R^n and by σ_0 its induced representation of the Lie algebra $\mathcal{L}$ of G on a vector space R^n. Let $\{\mathcal{E}_\rho\}$ be a base of $\mathcal{L} = R^n \otimes R^{n*}$. Taking a base $\{e_i\}$[1] of R^n we can express the representation σ of G on R^n by a matrix (g_{ik}):

(1) $$\sigma(g)e_k = g_{ik}e_i, \quad g = (g_{ik}) \in G.$$

Because $\sigma_0(\mathcal{E}_\rho)$ is an endomorphism of the vector space R^n, having chosen a base of R^n, we can write

(2) $$\sigma_0(\mathcal{E}_\rho)e_i = a^k_{i\rho}e_k,$$

where $(a^k_{i\rho})$ is a matrix, the elements of which belong to R, corresponding to $\mathcal{E}_\rho$. The adjoint representation of G on $\mathcal{L}$ be denoted, as usually, adj.

We have on G the left invariant vector fields

(3) $$\mathcal{E}_{ij}(g) = g_{ri}\left(\frac{\partial}{\partial x_{rj}}\right)_g, \quad g \in G.$$

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1) $i,j,k = 1,2,\dots,n$ ∥ $\rho,\sigma,\dots = 1,2,\dots,s$

It is easy to show that

(4) $$[\mathcal{E}_{ij}, \mathcal{E}_{rs}] = \delta_{si}\,\mathcal{E}_{rj} - \delta_{rj}\,\mathcal{E}_{is}\,.$$

The representation σ_0 of $\mathcal{L}$ on R^n can be expressed in the following way:

(5) $$\sigma_0\,(\mathcal{E}_{ij})e_r = \delta_{is}\,\delta_{jr}\,e_s\,.$$

If we have an R^n -valued q-form

(6) $$\varphi = \varphi^i \otimes e_i$$

on M and an $\mathcal{L}$ -valued q' -form

(7) $$\phi = \phi^\rho \otimes \mathcal{E}_\rho$$

on M , then we can define the R^n -valued $(q+q')$-form $\phi\cdot\varphi$ as follows:

(8) $$\phi\cdot\varphi = (\phi^\rho\otimes\mathcal{E}_\rho)\cdot(\varphi^i\otimes e_i) = \sigma_0\,(\mathcal{E}_\rho)\cdot e_i \otimes \phi^\rho \wedge \varphi^i\,.$$

Now let us have two vector spaces $A = R^n \otimes \overset{2}{\Lambda}\, R^n{}^*$, $B = \mathcal{L}\otimes R^{n*}$ (R^{n*} being the dual vector space of R^n). If we denote by σ^* the dual representation of σ , we obtain the representations $\mathcal{R} = \sigma \otimes \overset{2}{\Lambda}\,\sigma^*$ and $\mathcal{L} = adj \otimes \sigma^*$ of the group G on the vector spaces A and B respectively. Let $\{e^i\}$ be the dual base of the base $\{e_i\}$ of R^n . We define the linear map $\mathcal{A} : B \to A$ as follows:

(9) $$\mathcal{A}\,\Big(\sum_{\rho,k} \xi^\rho_k\, e^k \otimes \mathcal{E}_\rho\Big) = \sum_{\rho,i,j,k} (a^i_{\rho j}\,\xi^\rho_k - a^i_{\rho k}\,\xi^\rho_j)\; e_i \otimes e^j \wedge e^k.$$

Since we have $\mathcal{L} = R^n \otimes R^{n*}$, the vector $\mathcal{E}_\rho$ of the base $\{\mathcal{E}_\rho\}$ can be written in the form $\sum_{i,j} \sigma_0\,(\mathcal{E}_\rho) e_i \otimes e^j$ -in the base $\{e_i \otimes e^j\}$ of $R^n \otimes R^{n*}$.

Now we can write $\mathcal{E}_\rho = a^k_{i\rho}\, e_k \otimes e^i$. The mapping $\mathcal{A}$ is then, in fact, a mapping of $B = R^n \otimes R^{n*} \otimes R^{n*}$

into $A = R^n \otimes \overset{2}{\Lambda} R^{n*}$; which, having chosen a base of R^n, assigns to every element $s_{ij}^k e_k \otimes e^i \otimes e^j$ an element $(s_{ij}^k - s_{ji}^k) e_k \otimes e^i \wedge e^k$.

It is straightforward to verify that we have

(10) $\mathcal{A}\,\mathcal{L}(g) = \mathcal{R}(g)\,\mathcal{A}, \quad g \in G.$

If considering $x \in P$ as an isomorphism of R^n on $T_a(M)$, $(p x = a)$ we can define a fundamental 1-form as follows:

Definition: The fundamental 1-form on P is an R^n-valued 1-form θ on P, which assigns to a vector $\tau_x \in T_x(P)$ a vector

(11) $\theta(\tau_x) = x^{-1} \cdot p^{-1} \tau_x$.

It is easy to verify that a fundamental 1-form satisfies the following conditions:

a) $D_g^* \theta = g^{-1} \cdot \theta, \quad g \in G,$

b) $\theta(\tau) = 0 \iff p\tau = 0.$

Let a frame $\{\vartheta_\alpha^1, \dots, \vartheta_\alpha^n\}$ be given on a neighborhood U_α on M. The form $\vartheta_\alpha = \vartheta_\alpha^i \otimes e_i$ is an R^n-valued 1-form on U_α. If we consider two neighborhoods U_α, U_β; $U_\alpha \cap U_\beta \neq 0$ and if $g_{\alpha\beta}$ denotes the coordinate transformations of P, then $\vartheta_\alpha = g_{\alpha\beta}(a)\vartheta_\beta$, $a \in U_\alpha \cap U_\beta$. Denoting by p_α the cross-projection of P, it holds according to the definition of a principal fibre bundle that

$$p_\beta(x) = g_{\beta\alpha}(p(x))p_\alpha(x), \quad x \in p^{-1}(U_\alpha \cap U_\beta).$$

Now we can define the R^n-valued fundamental 1-form θ_α on P by $\theta_\alpha = p_\alpha^{-1}(x) \cdot p^* \vartheta_\alpha$. Namely, $\theta_\alpha = \theta_\beta$ holds.

A connexion Γ on P be given by a system of horizontal spaces. To every point $x \in P$ there is assigned a so-called horizontal space H_x and a so-called vertical space V_x so, that their union is $T_x(P)$. For arbitrary $g \in G$ and $x \in P$, $H_{xg} = D'g\, H_x$ and H_x depend differentiably on a point $x \in P$. A connexion Γ on P can also be given by an $\mathcal{L}$-valued differential 1-form ω on P. The R^n-valued 2-form on P

(12) $$\Sigma_\omega = d\theta + \omega\theta ,$$

where θ is a fundamental 1-form on P, is called the torsion form Σ_ω of a connexion Γ.

The tensor $t\Sigma_\omega$ assigned to the torsion form Σ_ω is called the torsion tensor. $t\Sigma_\omega$ is the mapping of P into A of the type $\mathcal{R}$.

2. In this part it will be shown that a differentiable transformation of certain type of P onto itself assigns to a connexion on P against a connexion on P and that there exists a tensor on P which depends only on a respective mapping of P onto itself.

Let h be a differentiable map of P onto itself, such that the following conditions are satisfied:

1) $h \circ D_g = D_g \circ h$

2) $h' \tau_x = \tau_{hx}$, for an arbitrary vertical vector τ_x.

3) If Γ is a connexion on P and τ_x a horizontal vector, then there exists a vertical vector v_x so,

that $h'\tau_x = \tau_{hx} + v_{hx}$.

A map $h'H_x$ of a vector space H_x is again a horizontal vector space at a point hx. Then let us denote H'_{hx}. It is easy to verify that a system of spaces H'_x which are assigned to the points of P, define a connexion Γ' on P. We have clearly

Proposition: Let ω and ω' denote the connexion forms of the connexions Γ and Γ' respectively. Γ' is the map of Γ as described above. Then the tensor tu, where $u = \omega' - \omega$, is independent on the choice of a connexion Γ on P.

Proof: Let $\{h_1, \cdots, h_n\}$ be a base of H_x and $\{v_1, \cdots, v_s\}$ be a base of V_x. Let $\tilde{h}_i$ be a map of a vector h_i, where the mapping h is chosen. $\{\tilde{h}_1, \cdots, \tilde{h}_n\}$ is obviously a base of H'_x (the horizontal space at the point $x \in P$ of the connexion Γ'). Hence we have

(13) $$h_i = \tilde{h}_i + v_i^\alpha v_\alpha ,$$

where v_i^α is a function of a point $x \in P$. A base $\{v_\alpha\}$ of V_x be chosen so, that the equation

(14) $$\omega(v_\alpha) = \varepsilon_\alpha$$

holds.

The tangent vector $\tau \in T_x(P)$ be given by

(15) $$\tau = a^i h_i + b^\alpha v_\alpha .$$

Now we have

(16) $$\omega(\tau) = b^\alpha \varepsilon_\alpha , \quad \omega'(\tau) = (b^\alpha + a^i v_i^\alpha)\varepsilon_\alpha ,$$

where ω' is the 1-form of the connexion Γ' on P.

By making use of (16), we have

(17) $u(\tau) = a^i v_i^\alpha \varepsilon_\alpha$.

If $\{h_1, \dots, h_n\}$ is the dual base of $\{\theta^1, \dots, \theta^n\}$ (θ^i are fundamental forms on M) we can write

(18) $u = v_i^\alpha \theta^i \otimes \varepsilon_\alpha$.

If $\{e^i \otimes e^j \otimes e_k\}$ is a base of B , then

(19) $a^k_{\varphi j} \; v_i^\rho$

are components of the tensor $t\,u$.

Let π be a connexion form of a new connexion Ω on P . The horizontal spaces K be formed by vectors

(20) $k_i = \lambda_i^j h_j + \varphi_i^\alpha v_\alpha$; $\lambda_i^j \; \varphi_i^\alpha$ are functions of a point $x \in P$. Assume that $Det\,|\lambda_i^j| \neq 0$ on P . If $a = (a_i^j)$, we shall denote by $(\tilde{a}_i^j) = a^{-1}$ the inverse of a . We can write now

(21) $h_i = \tilde{\lambda}_i^j k_j + \phi_i^\alpha v_\alpha$.

From the equations (19) and (20) we have

(22) $\phi_i^\alpha + \tilde{\lambda}_i^j \varphi_j^\alpha = 0$.

When K' is a h -map of the horizontal space K , we have the horizontal space K' spanned by vectors

(23) $\tilde{k}_i = \lambda_i^j \tilde{h}_j + \varphi_i^\alpha v_\alpha$.

We can write then

(24) $k_i = \tilde{k}_i + \lambda_i^k v_k^\alpha v_\alpha$.

The vector τ can be written as follows:

(25) $\tau = a^i \tilde{\lambda}_i^j k_j + (a^i \phi_i^\alpha + b^\alpha) v_\alpha$,

$$\tau = a^i \tilde{\lambda}_i^j \tilde{k}_j + (a^i \phi_i^\alpha + b^\alpha + a^k v_k^\alpha) v_\alpha$$

If we denote by π' the connexion form of the connexion Ω' (Ω' is the map of the connexion Ω) we have

(26) $\quad u' = \pi' - \pi = v_i^{\alpha}\, \theta^i \otimes \varepsilon_{\alpha}$.

From (18) and (26) we see that u is independent of the choice of the initial connexion. It is clear that the choice of a base of H does not play any essential role through the proof.

3. Throughout this chapter let us consider how to define the functions $v_i^{\alpha}(x)$ mentioned above. It is possible to show that having M_f associated to M, we can construct the functions $v_i^{\alpha}(x)$. In the neighborhood U_{x_0} of a point x_0 on M let us have a coordinate system $(x^1, \dots, x^n)$, $x_0 = (x_0^1, \cdots, x_0^n)$. The mapping f_{x_0} can be written as follows:

(27) $\quad x^{i'} = f^i_{x_0}(x^1, \dots, x^n),\ x_0^{i'} = x_0^i = f^i_{x_0}(x_0^1, \dots, x_0^n);\ (i = 1, 2, \dots, n).$

If we put

(28) $\quad A^i_k(x_0, x) = \dfrac{\partial f^i_{x_0}}{\partial x^k}$

We have

(29) $\quad A^i_k(x_0, x_0) = \delta^i_k$

because $f_{x_0} \in \mathcal{J}^1_{x_0} g$.

The function $A^i_k(x, y);\ x, y \in M$ can be considered as a function on $M \times M$, which is defined in some neighborhood V of the point $(x_0, x_0) \in M \times M$.
Let us have two neighborhoods $V_{\alpha}, V_{\beta};\ V_{\alpha} \cap V_{\beta} \neq \phi$ on $M \times M$ and let $(x^1, \dots, x^{2n})$, $(y^1, \dots, y^{2n})$ be coordinates of points $u \in V_{\alpha}$, $v \in V_{\beta}$ respectively, so that $(x^1, \dots, x^n)$, $(x^{n+1}, \dots, x^{2n})$ are coordinates in some neighborhoods U_1, U_2 on M and analogously $(y^1, \dots, y^n)$, $(y^{n+1}, \dots, y\, 2n)$ are coordinates in some

neighborhoods U_3, U_4 on M. For points of the intersection $V_\alpha \cap V_\beta$ let us have the same transformation of coordinates as it is for points of $U_2 \cap U_4$ on M. Namely, if we have a neighborhood U_1 on M, we can associate to every point $x \in U_1$ the neighborhood U_x on M. If we have $U_2 = \bigcup_{x \in U_1} U_x$ and analogously for U_3 and U_4, then we have $U_1 \subset U_2$, $U_3 \subset U_4$ and in that case it is sufficient to consider the transformations of coordinates in the intersection $U_2 \cap U_4$ only. In this interpretation $(x^1, \dots, x^n)$, $(x^{n+1}, \dots, x^{2n})$ are coordinates of two points of U_2.

We shall denote

$$(30) \qquad A^i_{k,j}(u) = \frac{\partial A^i_k (x^1, \dots, x^n, x^{n+1}, \dots, x^{2n})}{\partial x^{j+n}}, \quad (i,j,k = 1,2,\dots,n).$$

The functions are defined on V_α. Let $B^i_{kj}(v)$ be analogously defined functions on V_β. We obtain easily the relations

$$(31) \qquad B^i_k(v) = h^i_s(v)\, A^s_r(u)\, k^r_k(v); \quad h^i_s(v),\ k^r_k(v) \in G,$$

where $u = v$. It is straightforward to verify that

$$(32) \qquad B^i_{j,k}(v) = h^i_s(v)\, A^s_{r,e}(u)\, k^r_j(v)\, k^e_k(v) + (\cdots).$$

The expressions $(\cdots)$ are equal to zero on the diagonal $\widetilde{M}$ of the product $M \times M$. We shall not request those expressions in detail. On $\widetilde{M}$ there holds also:

$$(33) \qquad k^j_i(v) = \tilde{h}^j_i(v).$$

($\tilde{h}^j_i$ be the coefficient of the inverse matrix to the matrix (h^j_i)).

Let us identify the points $(v, w_\alpha) \in V_\alpha \times G$

$(v, w_\beta) \in V_\beta \times G, \quad v \in V_\alpha \cap V_\beta$ if the equation

(34) $$w_\alpha = h_{\alpha\beta}(v)\, w_\beta\,, \quad h_{\alpha\beta}(v) = (h^j_i(v))$$

is satisfied.

If we denote R the union of the sets $\{V_\alpha \times G\}$ under the above described identification, we have a principal fibre bundle R. We can consider R as a principal fibre bundle over $M \times M$, (if we shall make any extension of the covering and of the functions $h_{\alpha\beta}$) with the structural group G and natural projection $q : R \to M \times M$.

Let us define on $q^{-1}(V_\alpha)$ the functions

(35) $$\underset{(\alpha)}{v}{}^{ij}_{s}(w_\alpha) = A^e_{r|k}(u)\, \overset{(\alpha)}{w}_{ej}\, \overset{(\alpha)}{\tilde{w}}_{ir}\, \overset{(\alpha)}{\tilde{w}}_{sk}\,, \quad u = q(w_\alpha)\,.$$

If $u = v \in V_\alpha \cap V_\beta$, then we have

(36) $$\underset{(\beta)}{v}{}^{ij}_{s}(w_\beta) = B^e_{r|k}(v)\, \overset{(\beta)}{w}_{ej}\, \overset{(\beta)}{\tilde{w}}_{ir}\, \overset{(\beta)}{\tilde{w}}_{sk}\,.$$

Now, it follows from (31), (32), (33) that on $\tilde{M}$ the relation

(37) $$\underset{(\alpha)}{v}{}^{ij}_{s}(w_\alpha) = \underset{(\beta)}{v}{}^{ij}_{s}(w_\beta)$$

is satisfied. From (36) it follows that the functions (34) are globally defined on a principal fibre bundle $S(\tilde{M}, G)$ which is a submanifold of R. We can identify in a natural way $\tilde{M}$ and M. In that what follows we shall speak about the principal fibre bundle $P(M, G)$ instead of $S(\tilde{M}, G)$. Let us now define the map of P into $B = R^n \otimes R^{n*} \otimes R^{n*}$ as follows:

Let us associate an element

(38) $$v^{ik}_j(x)\, e_k \otimes e^i \otimes e^j$$

for the point $x \in P$. We can make now the identification

$$(39) \qquad v_j^{ik}(x) = a_{\rho j}^k \; v_i^\rho(x) \; ,$$

where $a_{\rho j}^k \; v_i^\rho(x)$ are the components of the tensor tu (19). From (5) we see that $a_{(ij)r}^s = \delta_{is}\,\delta_{jr}$ and that (38) can be written in the form

$$(40) \qquad v_j^{ik}(x) = a_{(rs)j}^k \; v_i^{(rs)}(x) = \delta_{rk}\;\delta_{js}\; v_i^{(rs)}(x) = v_i^{(kj)}(x).$$

The equation (13) goes under (39) into the equations

$$(41) \qquad h_i = \tilde{h}_i + v_i^{(kj)} v_{kj} = \tilde{h}_i + v_j^{ik}(x)\, v_{kj} \; , \; (i,j,k = 1,2,\dots,n)$$

If we have associated a combined manifold M_f to a manifold, then the differentiable mapping of connexions on P into itself is given. v_j^{ik} are the components of the tensor tu.

Definition: Let u be a $\mathcal{L}$-valued 1-form on P given in the chosen base by the equation (18). The functions $v_i^\alpha(x)$ be given by (38), (39). Then we say that the tensor tu is a linearisating tensor.

From (34) we get

$$(42) \qquad v_k^{ij} = v_j^{ki} \; .$$

According to the above considerations we have the following

Lemma : It is possible to associate a linearisating tensor tu on the pair $P(M, G)$ as a mapping of P into B to the combined manifold M_f. tu is the tensor corresponding to the 1-form on P defined by (18).

It is also possible to show that the following lemma is satisfied.

Lemma : Let ω be a Γ-connexion 1-form on P and ω' a Γ'-connexion 1-form on P, where Γ' is the map of Γ, when h is the mapping of P onto itself 2. Let Σ_ω, $\Sigma_{\omega'}$ be the torsion forms of these connexions respectively. Then the relation

$$(43) \qquad t\Sigma_\omega = t\Sigma_{\omega'}$$

satisfies their torsion tensors.

Proof. According to the definition (12) of the torsion form we can write

$$(44) \qquad \begin{aligned} \Sigma_{\omega'} - \Sigma_\omega &= u \cdot \theta = \\ &= (v_i^\alpha \theta^i \otimes \mathcal{E}_\alpha) \cdot (\theta^k \otimes e_k) = \\ &= v_i^\alpha \sigma_o(\mathcal{E}_\alpha) e_k \otimes \theta^i \wedge \theta^k = \\ &= a_{\alpha k}^j v_i^\alpha e_j \otimes \theta^i \wedge \theta^k = \\ &= \frac{1}{2}(a_{\alpha k}^j v_i^\alpha - a_{\alpha i}^j v_k^\alpha) e_j \otimes \theta^i \wedge \theta^k \end{aligned}$$

But from (41) we have

$$(45) \qquad a_{\alpha k}^j v_i^\alpha = a_{\alpha i}^j v_k^\alpha$$

and then we have

$$(46) \qquad t\Sigma_{\omega'} - t\Sigma_\omega = \mathcal{A} \cdot tu = 0 .$$

4. If we compare the notion of linearisating tensor introduced above with that one defined by E. Čech [3], then we have

Proposition: Let $M \equiv A_n$ be an n-dimensional affine space and f_x be the tangent mapping of the identity mapping of A_n onto itself at a point $x \in A_n$. Then the linearisating tensor associated to $(A_n)_f$ is the Čech's linearisating tensor [3].

Proof: Let $\{A, \mathcal{I}_1, \dots, \mathcal{I}_n\}$ be a moving frame of A_n. We have the well known fundamental equations

(47) $$dA = \omega^i \mathcal{I}_i\ ,\ d\mathcal{I}_i = \omega_i^k \mathcal{I}_k,\ (i,\ k = 1, 2, \dots, n).$$

Let $\mathbb{A}$ be an affine mapping of A_n onto itself. Let $\{A', \mathcal{I}'_1, \dots, \mathcal{I}'_n\}$ be a $\mathbb{A}$-map of $\{A, \mathcal{I}_1, \dots, \mathcal{I}_n\}$.

(48) $$\mathbb{A} A = A'\ ,\ \mathbb{A}\mathcal{I}_i = \mathcal{I}'_i \quad (i = 1, 2, \dots, n).$$

According to (46) we have

(49) $$dA' = \pi^i \mathcal{I}'_i,\ d\mathcal{I}'_i = \pi_i^k \mathcal{I}'_k\ .$$

We say that the mapping $\mathbb{A}$ is a tangent affine mapping if the following conditions are satisfied:

(50) $$\mathbb{A} A = A',\ \mathbb{A} dA = dA',\ A = A'\ .$$

The necessary and sufficient conditions for $\mathbb{A}$ to be a tangent affine mapping are:

(51) $$\omega^i = \pi^i \quad (i = 1, 2, \dots, n).$$

We have then

(52) $$[\omega^k \omega_k^i - \pi_k^i] = 0 \quad (i, k = 1, 2, \dots, n).$$

According to (51) we have

(53) $$\omega_k^i - \pi_k^i = c_{kj}^i\ \omega^j \quad (i, j, k = 1, 2, \dots, n),$$

where $c_{kj}^i = c_{jk}^i$. c_{jk}^i is a tensor defined on the neighborhood of a point A. It is so called Čech's linearisating tensor which plays a fundamental role in the theory of correspondences between two affine (projective,...) spaces. That this tensor is a linearisating tensor mentioned above it is straightforward to see from [2].

R e f e r e n c e s

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