Jiří Jarušek

# On the regularity of solutions of a thermoelastic system under noncontinuous heating regimes. II

*Applications of Mathematics*, Vol. 36 (1991), No. 3, 161–180

Persistent URL: http://dml.cz/dmlcz/104457

# ON THE REGULARITY OF SOLUTIONS OF A THERMOELASTIC SYSTEM UNDER NONCONTINUOUS HEATING REGIMES. PART II

JIŘÍ JARUŠEK, Prague

(Received July 13, 1988)

*Summary.* A quasilinear noncoupled thermoelastic system is studied both on a threedimensional bounded domain with a smooth boundary and for a generalized model involving the influence of supports. Sufficient conditions are derived under which the stresses are bounded and continuous on the closure of the domain.

*Keywords:* Quasilinear heat equation, Lamé system, noncontinuous heating regimes, Sobolev spaces, Fourier transformation, supports, boundedness and continuity of the stresses with respect to space variables and in time.

*AMS classification:* 73U05 (35B65, 35M05, 35R05).

## 0. INTRODUCTION

In [5] we proved the global boundedness and continuity of the thermoelastic stress for a twodimensional model of a heated body, where the heating equation was quasilinear with nonlinear boundary value conditions and the linear Lamé system was considered. The heating regimes could be noncontinuous both in the space variable and in time (only the monotonicity or bounded variation in time was assumed). Also the influence of isolated boundary nonsmoothness was studied for the model.

In the present paper we will extend the results. In Sec. 1 we study the threedimensional case for a body with a sufficiently smooth boundary. We preserve the noncontinuity of the heating regime in time, but at least in one space variable we need a sufficiently high differentiability. In Sec. 2 we study the influence of the boundary singularities, particularly at the points of change of the boundary value condition which describe the influence of the supports of the heated body on the stress.

We recall the origin of the problem being in technical practice – the original problem has all coefficients in the heat equation temperature-dependent – and consider a boundary value condition of the Stefan-Boltzmann type. The proved boundedness of the stresses indicates the suitability of the model (in the described situations). The non-existence of the phase transition enables us to transform the original problem to the following system of partial differential equations, where we look for the temperature $u$ and the displacement $v$:

$$
(1) \qquad \begin{cases} \beta(u) \dfrac{\partial u}{\partial t} = \Delta u \quad \text{on} \quad Q = (0, \mathscr{T}) \times \Omega \,, \\ \dfrac{\partial u}{\partial \nu} = g(T) - g(u) \quad \text{on} \quad S = (0, \mathscr{T}) \times \Omega \,, \quad u(0, \cdot) = u_0 \equiv 0 \quad \text{on} \quad \Omega \,; \end{cases}
$$

$$
(2) \qquad \begin{cases} (1 - 2\sigma) \Delta v + \nabla \operatorname{div} v = (2 + 2\sigma) \nabla \gamma(u) \quad \text{on} \quad \Omega \quad \text{for each} \quad t \in (0, \mathscr{T}) \,, \\ (1 - 2\sigma)\left(\dfrac{\partial v}{\partial \nu} + ((\nu, \nabla_i v)_i)\right) + 2\sigma \nu \operatorname{div} v = (1 + 2\sigma) \gamma(u) \, \nu \quad \text{on} \quad \partial\Omega \\ \text{for each} \quad t \in (0, \mathscr{T}) \,. \end{cases}
$$

We remark that we avoid introducing the "transforming" one-to-one function $\Lambda$ (we transform also $T$). $\Omega \subset R^N$ is a bounded domain occupied by the heated body having the boundary $\partial\Omega$, $T$: $S \to R$ is a non-negative heating regime with $T(0, \cdot) \equiv \equiv 0$ on $\partial\Omega$, $\beta$ is a nondecreasing strictly positive function, i.e. there is a constant $\beta_0 > 0$ such that $\beta \geqq \beta_0$, $\gamma$: $R^1 \to R^1$ is a sufficiently smooth function, $g$: $R^1 \to R^1$ (or $g$: $\Gamma \times R^1 \to R^1$ in Sec. 2) is nondecreasing with $g(0) = 0$. The appropriate requirements concerning the differentiability of $\beta$, $g$ will be specified in the corresponding parts of the sections. $\nabla$ denotes the gradient with respect to the space variables, $\nu$ the unit outer normal vector (at the points of $\partial\Omega$), $\sigma$ is a constant from $(0, \frac{1}{2})$ (the Poisson ratio). For a sufficiently smooth function $E$ (the Young modulus of elasticity) we define the stress tensor

$$
(3) \qquad \tau_{ij} = \frac{E(u)}{(2 + 2\sigma)(1 - 2\sigma)} \left( (1 - 2\sigma) \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \delta_{ij} \, 2\sigma \sum_{k=1}^{N} \frac{\partial v_k}{\partial x_k} \right),
$$

$$
i, j = 1, \ldots, N = \dim \Omega \,,
$$

$$
\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}
$$

which corresponds to the thermoelastic stress. For a domain $M \subset R^m$ and a non-negative vector $\alpha \in R^m$ we denote by $H^\alpha(M)$ the anisotropic Sobolev space (with fractional derivatives if there is $i$ for which $\alpha_i$ is non-integer) of the Hilbert type ($\alpha_i$ is the order of the square integrable derivative in the i-th direction, $i = 1, \ldots, m$). For the definition cf. [1], [5], ...

## 1. THE THREEDIMENSIONAL CASE WITH LINEAR HEATING EQUATION AND NONLINEAR BOUNDARY VALUE CONDITION

Under the above mentioned suppositions for $\partial\Omega$ of the class $C_{0,1}$ and $T$ non-decreasing in time and bounded, it is possible to prove in the same way as in Thm. 1 of [12] and Prop. 3 of [5] that $u \in L_\infty(Q)$ and $\partial u/\partial t \in L_2(Q)$. The monotonicity of $T$ could be replaced by its uniformly (on $\partial\Omega$) bounded variation in time. If moreover $T \in L_2(0, \mathscr{T}; H^\alpha(\partial\Omega))$, $\alpha \in (0, \frac{1}{2}\rangle$, then it is possible to prove as in Prop. 4 of [5] (using the regularity results for linear elliptic equations) that $u \in H^{1,3/2+\alpha}(Q)$ (the first number concerns the generalized differentiability in time, the second in the space variables). Under the supposition of analyticity of $\beta$ we are able to prove for a monotone $T$ that $u \in \bigcap_{\varepsilon>0} C_0(0, \mathscr{T}; H^{1+\tilde{\alpha}-\varepsilon}(\Omega))$, $\tilde{\alpha} = (1 + 2\alpha)^2/(14 + 12\alpha)$. The same result can be proved for $T$ with uniformly bounded variation, but for $\beta = \beta_0$, a constant. In 3 dimensions, however, such result does not yield continuity of $u$ on $\bar{Q}$ even for $\alpha = 1/2$ for which $\tilde{\alpha}(\alpha) = 1/5$. The proof of continuity of $u$ via the imbedding theorem has the main role in the proof of continuity and boundedness of the stress.

We recall the variational formulation of (1), where on $(-\infty, 0)$ we can suppose $u \equiv 0$, $T \equiv 0$,

$$(4) \qquad \int_{-\infty}^{t_0} \int_\Omega \beta(u) \frac{\partial u}{\partial t} w \, dx \, dt + \int_{-\infty}^{t_0} \int_\Omega \nabla u \nabla w \, dx \, dt + \int_{-\infty}^{t_0} \int_{\partial\Omega} \mathscr{g}(u) w \, dx \, dt =$$

$$= \int_{-\infty}^{t_0} \mathscr{g}(T) w \, dx \, dt, \quad t_0 \leqq \mathscr{T}, \quad w \in H^1(Q).$$

To obtain the required regularity of the stress tensor, we have to prove higher regularity of $u$. There is small hope of obtaining better regularity in time and therefore it is difficult to improve the regularity in the normal direction to the boundary if the noncontinuity of $T$ in time is supposed. Therefore it seems that the only reserve is in the tangential directions of the space variables, which naturally needs higher space differentiability of $\mathscr{g}(T)$. In the proof we will employ the method of shifts in arguments and higher order differences. Therefore we suppose that $\beta \equiv \beta_0$, a positive constant. $\sup_S T$ will be denoted by $\bar{B}$ which is also the upper bound for $u$ being, of course, non-negative.

In the rest of the section we suppose sufficient smoothness of $\partial\Omega$ and $\gamma$. For the proof of better tangential regularity we again employ the technique of the local straightening of the boundary. Let $x_0 \in \partial\Omega$ be arbitrary. We suppose that $x_0 = 0$ and in a neighbourhood $\mathscr{U}$ of $x_0$, $\partial\Omega$ is described by a sufficiently smooth function $\varphi: R^2 \to R^1$ such that $\varphi(0) = 0$, $\nabla\varphi(0) = 0$ and $\Omega \cap \mathscr{U} = \{x \in R^3; x_3 > \varphi(x_1, x_2)\}$. Such a situation can be ensured by a suitable rotation and shift of the coordinate system. The local straightening of the coordinates has the form $[x_1, x_2, x_3] \mapsto$ $\mapsto [x_1, x_2, x_3 - \varphi(x_1, x_2)]$. Extending $T$ onto $(\mathscr{T}, +\infty) \times \partial\Omega$ by $T(t, x) =$

$= \omega(t)\, T(\mathscr{T}, x)$, $t > \mathscr{T}$, where $\omega\colon R^1 \to \langle 0, 1 \rangle$ is a $C_2$-smooth function such that $\omega((-\infty, 4\mathscr{T})) = \{1\}$, $\omega((5\mathscr{T}, +\infty)) = \{0\}$ and $\omega' < 0$ on $R^1$ and using $\varrho_0 \in C_1(R^1)$ such that $\varrho_0((-\infty, 2\mathscr{T})) = \{1\}$, $\varrho_0((3\mathscr{T}, +\infty)) = \{0\}$ and $\varrho_0' \leqq 0$ on $R^1$ we can extend (4) for particular test functions to $t_0 \in R^1$. Let $B_\delta(x_0)$ be the $\delta$-ball with the center $x_0$ such that $\delta < 1/3 \operatorname{dist}(x_0, R^3 \setminus \mathscr{U})$ and $|\nabla \varphi| < \tilde{\varepsilon}_0$ for a given small $\tilde{\varepsilon}_0 > 0$, let $\bar{\varrho} \in \mathfrak{R}_\delta$ (a sufficiently smooth partition of unity on $\Omega$ with sufficiently small diameters of supports – cf. [4]) such that $\bar{\varrho}(x_0) \neq 0$. After the straightening of the boundary (4) will be transformed for $w = \varrho^2 \omega$, $\varrho = \varrho_0 \bar{\varrho}$, $w \in H^1(Q)$ into the equality

$$\text{(5)} \qquad \int_{-\infty}^{t_0} \int_{R^2 \times (0,\delta)} \beta_0 \frac{\partial u}{\partial t} \varrho^2 \omega + (\nabla u, \nabla \varrho^2 \omega)_3 + \\ + \mathscr{B}(u, \varrho^2 \omega)\, \mathrm{d}x\, \mathrm{d}t + \int_{R^2} \int_{-\infty}^{t_0} g(u)\, \varrho^2 \omega J\, \mathrm{d}x\, \mathrm{d}t = \\ = \int_{-\infty}^{t_0} \int_{R^2} g(T)\, \varrho^2 \omega J\, \mathrm{d}x\, \mathrm{d}t\,, \quad t_0 \in R\,,$$

where the perturbation form $\mathscr{B}$ emerging from the straightening of the boundary has small coefficients (its magnitude depends on $\tilde{\varepsilon}_0$) and

$$J(x_1, x_2) = \sqrt{\left(1 + \sum_{i=1}^{2} \left(\frac{\partial \varphi}{\partial x_i}(x_1, x_2)\right)^2\right)}\,.$$

Denoting by $(5)_{-h}$ the shifted equality (5) in the direction $h \in R^2 \times \{0\}$ we take $(5)_{-2h} - 2(5)_{-h} + (5)$ and put $\omega = \omega_{-h} = \omega_{-2h} = \varrho^2 \Delta_2^h u$, where

$$\text{(6)} \qquad \Delta_k^h f \equiv \sum_{j=0}^{k} (-1)^j \binom{k}{j} f_{-(k-j)h}\,.$$

The resulting equality will be multiplied by $|h|^{-2-2\alpha}$, $\alpha > 1$, and integrated over $R^2$. We obtain an equality which we denote without writing as (7).

From (7) the natural energy-type estimate will be derived. On the left-hand side of the equality we have the terms to be estimated, namely

$$\text{(8)} \qquad \int_{R^2} |h|^{-2-2\alpha} \int_{R^3 \times (0,\delta)} (\nabla \Delta_2^h \varrho u)^2\, \mathrm{d}x\, \mathrm{d}t\, \mathrm{d}h + \\ + \sup_{t \in R^1} \frac{\beta_0}{2} \int_{R^2} |h|^{-2-2\alpha} \int_{R^2 \times (0,\delta)} (\Delta_2^h \varrho u)^2 (t, x)\, \mathrm{d}x\, \mathrm{d}h\,.$$

The other terms will be estimated on the right hand side. To do it, we exploit the fact that

$$\text{(9)} \qquad \varrho\, \Delta_k^h w = \Delta_k^h(\varrho w) - \sum_{l=1}^{k} \binom{k}{l} \Delta_l^h \varrho\, \Delta_{k-l}^h (w_{-lh})\,,$$

the Lipschitz continuity of $g$, $\varrho$, $J$, of the coefficients of the form $\mathscr{B}$ and of the first derivatives of all the functions mentioned. In this way we are able to estimate all the remaining volume integral terms by means of $\|u\|_{H^{1,2}}(Q)$ and with the help of $\|u\|_{L_\infty(Q)}$ or by means of a strong norm whose finiteness is proved in the steps of the proof preceding the step just executed. The only problematic terms are the boundary terms containing nonlinearities. First, we will formulate our results provided $G = \mathscr{g}(T)$. Thus we must be careful only when estimating the term

$$(10)\qquad \mathscr{J} \int_{\varphi_\delta} |h|^{-2-2\alpha} \int_{R^3\times\{0\}} (\Delta_2^h(\varrho \mathscr{g}(u)))\,(\Delta_2^h(\varrho u))\,\mathrm{d}x\,\mathrm{d}t\,\mathrm{d}h \leqq$$
$$\leqq \Big(\int_{\varphi_\delta} |h|^{-2-2\beta_1} \int_{R^3\times\{0\}} (\Delta_2^h(\varrho u))^2\,\mathrm{d}x\,\mathrm{d}t\,\mathrm{d}h\,.$$
$$\cdot \int_{\varphi_\delta\times R^3\times\{0\}} |h|^{-2-2(2\alpha-\beta_1)}\,(\Delta_2^h(\varrho \mathscr{g}(u)))^2\,\mathrm{d}x\,\mathrm{d}t\,\mathrm{d}h\Big)^{1/2}\,,$$
$$\mathscr{J} = \sup_{\varphi_\delta} J\,,$$
$$\varphi_\delta = \langle -\delta, \delta\rangle^2 \quad \text{and} \quad \alpha, \beta_1, 2\alpha - \beta_1 \in (0, 2)\,.$$

We remark that in the two-dimensional case we could restrict ourselves to the use of the first-order differences which together with the monotonicity of $\mathscr{g}$ enable us to estimate the term corresponding to (10) on the left hand side of the resulting inequality. Since the regularization procedure (cf. (15) below) and the trace theorem (cf. Prop. 2 below or Theorem of [5]) imply

$$\varrho u|_{R^3\times\{0\}} \in \bigcap_{\varepsilon>0} H^{(3/4)-\varepsilon,(3/2)-\varepsilon}(R^3 \times \{0\})\,, \quad \text{we put} \quad \beta_1 = \tfrac{3}{2} - \varepsilon\,, \quad \varepsilon > 0$$

can be arbitrarily small. To deal with the second integral on the right-hand side of (10) we recall the following propositions, where $\hat{f}$ denotes the Fourier transform of $f$ and $R^1_+ = (0, +\infty)$.

**Proposition 1.**

$$\int_{R^N}\int_{R^N} \left(\frac{\Delta_k^h f(x)}{|h|^{N/2+\alpha}}\right)^2 \mathrm{d}h\,\mathrm{d}x = c_{N,k}(\alpha) \int_{R^N} |\hat{f}|^2\,|\xi|^{2\alpha}\,\mathrm{d}\xi\,,$$

*where $N \geqq 1$ and $k \geqq 1$ are arbitrary integers, $\alpha \in (0, k)$ arbitrary and $f$ an arbitrary measurable function on $R^N$. $c_{N,k}(\alpha)$ does not depend on $f$.*

Proof. Cf. Lemma 1 of [5], where $c_{N,k}(\alpha)$ are calculated. □

**Corollary.** *For an arbitrary couple $f_1, f_2 \in H^\alpha(R^N)$, $\alpha > 0$, the integers $k_1, k_2, k_3, l$ such that $k_1, k_2 > \alpha > l$ and $k_3 > \alpha - l$, we have*

$$\int_{R^N}\int_{R^N} \frac{(\Delta_{k_1}^h f_1(x))\,(\Delta_{k_1}^h f_2(x))}{|h|^{(N/2+\alpha)2}}\,\mathrm{d}h\,\mathrm{d}x =$$
$$= \frac{c_{N,k_1}(\alpha)}{c_{N,k_2}(\alpha)} \int_{R^N}\int_{R^N} \frac{(\Delta_{k_2}^h f_1(x))(\Delta_{k_2}^h f_2(x))}{(|h|^{N/2+\alpha})^2}\,\mathrm{d}h\,\mathrm{d}x =$$
$$= \frac{c_{N,k_1}(\alpha)}{c_{N,k_3}(\alpha - l)} \int_{R^N}\int_{R^N} \frac{(\Delta_{k_3}^h \nabla^l f_1(x))\,(\Delta_{k_3}^h \nabla^l f_2(x)}{(|h|^{N/2+\alpha-l})^2}\,\mathrm{d}h\,\mathrm{d}x\,.$$

Proof. We apply Prop. 1 to $f_1$, $f_2$ and $f_1 + f_2$ for $k_1$, $k_2$ and to $\nabla^l f_1$, $\nabla^l f_2$, $\nabla^l(f_1 + f_2)$ for $k = k_3$. $\square$

**Proposition 2.** (Cf. [1].) *Let $\Omega$ be a domain in $R^N$ with a sufficiently smooth boundary $\partial\Omega$. Let $a = [a_1, \ldots, a_N] \in \bar{R}^N_+$ with at most one non-integer component and $\alpha = [\alpha_1, \ldots, \alpha_N] \in R^N_+$, $p \in (1, +\infty)$. Then $D^a f \in L_p(\Omega)$ if $f \in H^\alpha(\Omega)$ and*

$$(*) \qquad \sum_{i=1}^{N} \frac{1}{\alpha_i}\left(\frac{1}{2} - \frac{1}{p} + a_i\right) < 1 .$$

*If (*) holds with $1/p = 0$, then $D^a f \in C_0(\Omega)$.*

The proof for $\Omega = R^N$ follows from the one-dimensional case of Thm. 18.4 of [1] via Prop. 1 and the mathematical induction in $N$.

Using Proposition 1, we can see that it suffices to estimate

$$(11) \qquad \mathscr{I} = \frac{1}{\eta}\int_{\mathscr{I}\delta}\int_{R^3\times\{0\}} (\Delta_1^h(\tilde{\nabla}(\varrho\, g(u))), \Delta_1^h(\tilde{\nabla}(\varrho\, g(u))))_2\, |h|^{-2(2\alpha-\beta_1)}\, dx\, dt\, dh\,,$$

$$\tilde{\nabla} = \left[\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}\right],$$

$\eta > 0$ sufficiently small,

because we use the Hölder inequality with $1/\eta$ for the first and with $\eta$ for the second integral on the right-hand side of (10), with $\eta > 0$ arbitrarily small. Clearly,

$$(12) \qquad \Delta_1^h(\tilde{\nabla}(\varrho\, g(u))) = g'(u_{-h})\, \Delta_1^h(\varrho\,(\tilde{\nabla}u)) + (\tilde{\nabla}u)\,(\Delta_1^h(\varrho\, g'(u))) + \Delta_1^h\,(g(u)\,\tilde{\nabla}\varrho)\,.$$

The last term in (12) is, of course, unimportant. We have to estimate

$$(13) \qquad \frac{1}{\eta}\,\text{const}\,\Big(\int_{\mathscr{I}\delta}\int_{R^3\times\{0\}} (g'(u))^2\,(\Delta_1^h\,\tilde{\nabla}(\varrho u), \Delta_1^h\,\tilde{\nabla}(\varrho u))_2\, |h|^{-2(2\alpha-\beta_1)}\, dx\, dt\, dh\, +$$
$$+ \sup_{y\in\langle 0,B\rangle} |g''(y)|)^2 \int_{\mathscr{I}\delta}\int_{R^3\times\{0\}} |\tilde{\nabla}u|^2\,(\Delta_1^h(\varrho u))^2\, |h|^{-2(2\alpha-\beta_1)}\, dx\, dt\, dh\Big)\,,$$

$0 < \eta$ sufficiently small,

because $|\Delta_1^h\, g'(u)| \leqq \sup_{y\in\langle 0,\bar{B}\rangle} |g''(y)|\, |\Delta_1^h u|$. The first term in (13) can be estimated for $\alpha < 3/2$ by $\|u\|^2_{H^{1,2}(Q)} \sup_{y\in\langle 0,\bar{B}\rangle} |g'(y)|)^2$ . const. As $\beta_1$ can be chosen to equal $3/2 - \varepsilon$, $\varepsilon > 0$ arbitrarily small, the exponent at $|h|$ in the second integral term in (13) is greater than $-2$ for every $\alpha < 5/4$ and the integral term can be estimated by $4\bar{B}^2\|\nabla u\|^2_{L_2(Q,R^3)}$ for such $\alpha$. Thus, for $\alpha < 5/4$ we can estimate all nonlinear terms. Since on the left hand side of the energy inequality we have estimated $\int_{R^5\times(0,r)}$ . . $|h|^{-2-5/2+\varepsilon}\,(\Delta_2^h U, \Delta_2^h U)_2\, dx\, dh\, dt$, where $U$ denotes $\varrho u$ and $\varepsilon > 0$ is arbitrarily small, we have in fact proved that

$$(14) \qquad U \in \bigcap_{\varepsilon>0} H^{1,9/4-\varepsilon,9/4-\varepsilon,2}(R^3 \times (0, \delta))\,,$$
$$U/_{R^3\times\{0\}} \in \bigcap_{\varepsilon>0} H^{3/4-\varepsilon,7/4-\varepsilon,7/4-\varepsilon}(R^3 \times \{0\})\,,$$

where the indices denote the existence of square integrable derivatives of the indicated fractional orders in time, in the tangetial space variables and in the normal space va-riable, respectively. The second assertion is a consequence of the inequality

$$(15) \qquad |\xi_i|^{1+2\alpha-\varepsilon}\,|\xi_3|^{1+\varepsilon_0} \leqq \mathscr{K}(\alpha, \varepsilon_0, \varepsilon)\,(|\xi_3|^2\,|\xi_i|^{2\alpha} + |\xi_i|^{2+2\alpha})\,,$$
$$i = 1, 2, \quad \varepsilon(\varepsilon_0) \searrow 0 \quad \text{for} \quad \varepsilon_0 \searrow 0\,,$$

applied to the Fourier transform of $U$ extended to $R$ e.g. by means of the procedure described in Chapter 1 of [9].

Supposing a sufficient regularity of the input data we will continue the regularization procedure. Let $U/_{R^3\times\{0\}} \in \bigcap_{\varepsilon>0} H^{3/4-\varepsilon,3/2+a_0-\varepsilon,3/2+a_0-\varepsilon}(R^3 \times \{0\})$, $a_0 \in (0, 1)$ and $U \in \bigcap_{\varepsilon>0} H^{1,2+a_0-\varepsilon,2+a_0-\varepsilon,2}(R^3 + (0, \delta))$ for the same $a_0$ for every $\bar{\varrho} \in \mathfrak{R}_\delta$. To estimate the first (linear) term in (10), we can use the norm of the "new" space (we put $\beta_1 = \frac{3}{2} + a_1$) i.e. of $H^{3/4-\varepsilon,3/2+a_1-\varepsilon,3/2+a_1-\varepsilon}(R^3 \times \{0\})$ with $\varepsilon > 0$ arbitrarily small, because $\eta$ in (11) is small. Thus we can take $\beta_1 = \frac{3}{2} + a_1 - \varepsilon$ ($\varepsilon > 0$ arbitrarily small) in (10), (11), (13). Applying Prop. 1 and its corollary to (10), we avoid the necessity of supposing $\beta_1 < 2$. To estimate the first integral in (13) we need only $2\alpha - \beta_1 \approx a_1 + \frac{1}{2} < \frac{3}{2} + a_0$, i.e. $a_1 < 1 + a_0$. The second integral in (13) can be estimated by means of the Hölder inequality as follows

$$(16) \qquad \left(\int_{\mathscr{g}\delta} \int_{R^1\times 2\mathscr{g}\delta\times\{0\}} |\tilde{\nabla} u|^{2p}\,|h|^{-2+\varepsilon_1}\,\mathrm{d}h\,\mathrm{d}x\,\mathrm{d}t\right)^{1/p}.$$
$$\left(\int_{\mathscr{g}\delta} \int_{R^3\times\{0\}} |h|^{-2}\,(|h|^{-a_1+1/2-\varepsilon_2}\,\Delta_1^h U)^{(2p)/(p-1)}\,\mathrm{d}x\,\mathrm{d}h\,\mathrm{d}t\right)^{(p-1)/p},$$
$$\varepsilon_2 > \varepsilon_1 > 0\,.$$

Now, we use Prop. 2 to estimate (16) by the "old" $H^{3/4-\varepsilon,3/2+a_0-\varepsilon,3/2+a_0-\varepsilon}(R^3 \times \{0\})$-norm, $\varepsilon > 0$ arbitrarily small. From the estimate of the first term of (16) – to establish it, we can change if necessary the parameter $\delta$, the maximal diameter of the support of the partition of unity – we obtain $1/p > (9 - 2a_0)/(12 + 4a_0)$. The estimate of the second term yields $a_1 < a_0 + 2 - (2/p)(1 + a_0/3)$, i.e. $a_1 < \frac{1}{2} + \frac{4}{3}a_0$. Simultaneously the exponent at $|h|$ multiplying $\Delta_1^h U$ in the second integral of (16) must be greater than $-1$ (cf. [1]), i.e. $a_1 < \frac{3}{2}$. Repeating the procedure, we obtain recursively a sequence $\{\tilde{a}_i\}$ as follows

$$(17) \qquad \tilde{a}_{i+1} = \min\left(\tfrac{1}{2} + \tfrac{4}{3}a_i, \tfrac{3}{2}\right), \quad \tilde{a}_0 = \tfrac{1}{4}\,.$$

As $\tilde{a}_1 = \frac{5}{6}$, $\tilde{a}_2 = \frac{3}{2}$, after 2 steps and under a sufficient regularity of $\mathscr{g}(T)$, we easily prove that

$$U \in \bigcap_{\varepsilon>0} H^{1,7/2-\varepsilon,7/2-\varepsilon,2}(R^3 + (0, \delta)),$$
$$U/_{R^3} \times \{0\} \in \bigcap_{\varepsilon>0} H^{3/4-\varepsilon,3-\varepsilon,3-\varepsilon}(R^3 \times \{0\})$$

for every $\bar{\varrho} \in \mathfrak{R}_\delta$.

From the Hölder inequality

$$(18)\qquad |\tau^{1+\varepsilon_0}\xi_i^{\eta_i-\varepsilon}| \leqq |\tau|^2 + |\xi|^{2\eta_i-\varepsilon_0}\,, \quad i = 1, 2, 3\,, \quad \varepsilon(\varepsilon_0) \searrow 0 \quad \text{for} \quad \varepsilon_0 \searrow 0$$

and the $C_0$-imbedding theorem (cf. Prop. 2 or Theorem 1 of [2]), we obtain that

$$U \in \bigcap_{\varepsilon>0} C_0(R^1; H^{\tilde{\alpha}-\varepsilon,\tilde{\alpha}-\varepsilon,1-\varepsilon}(R^2 \times (0, r)))$$

$$\text{for} \quad U \in H^{1,2\tilde{\alpha},2\tilde{\alpha},2}(R^3 \times (0, r))\,.$$

Confronting the present result with the result mentioned at the beginning of this section, we obtain

$$U \in \bigcap_{\varepsilon>0} C_0(R^1; H^{\tilde{\alpha}-\varepsilon,\tilde{\alpha}-\varepsilon,6/5-\varepsilon}\ (R^2 \times (0, \delta))) \quad \text{for} \quad \tilde{\alpha} > \tfrac{6}{5}\,.$$

**Proposition 3.** *If the regularization procedure can be proceed up to* $\frac{17}{7} + 2\vartheta_0$, $\vartheta_0 > 0$ *arbitrarily small, we obtain the stress tensor* $\tau \in C_0(\bar{Q}; R^\sigma)$.

Proof. In this case $\tilde{\alpha} = \frac{12}{7} + \vartheta_0$. It suffices to prove that $\varrho\,\gamma(u)$ belongs also to $\bigcap_{\varepsilon>0} C_0(0, \mathscr{T}; H^{12/7+\vartheta_1,12/7+\vartheta_1,6/5-\varepsilon}\ (R^2 \times (0, \delta)))$ for some $\vartheta_1 > 0$, because then we can use Prop. 2 of [5] and prove that the first derivatives of the displacement $v$ have the same properties. Using the continuity of $E$ and Prop. 2 from this section we arrive at the desired result.

To prove the required regularity of $\varrho\,\gamma(u)$ we apply the extension technique of [9] and use (11) for the volume integral (over $R^3$ in $x$ and $h$), the exponent at $|h|$ will be $-2\alpha$ and the function $g$ is replaced by $\gamma$. We use (12) and arrive at (13) with the same modification as in (11). For the normal regularity we have $a = \alpha - 1 < \frac{1}{6}$. The estimate of the first term of the modified formula (13) is easy, for the second term we use the technique as in (16) in combination with Prop. 2. The estimate of such a modified expression (16) implies $1/p > 1/6$ for the first term, the estimate of the second term gives $a < \frac{6}{5}(1 - 1/p)$, i.e. $a < \frac{1}{5}$. Together with the preceding estimate we prove the square integrability of all normal derivatives of $\varrho\,\gamma(u)$ of the order $\frac{6}{5} - \varepsilon$, $\varepsilon > 0$ arbitrary. For the tangential case we have $a = \alpha - 1 = \frac{5}{7} + \vartheta_1$. We put $\frac{12}{7} + \vartheta_0 = \frac{12}{7}\,[1/(1 - k)]$, $k > 0$, because $\vartheta_0 > 0$, and proceed as above. The only difficulty is the estimation of the term corresponding to (16). Prop. 2 yields $1/p > (7 - 14k)/(12 - 7k)$ for its first part, for the second part it gives $a < (\frac{5}{7} + 2k)/(1 - k)$. Thus we can easily find $\vartheta_1 > 0$ such that the tangential derivatives of $\varrho\,\gamma(u)$ of the order $\frac{12}{7} + \vartheta_1$ are square integrable. □

To obtain the explicit dependence of the continuity of $\tau/_{\bar{Q}}$ on the input data, we shall carefully estimate the term

$$(19)\qquad \int_{R^5} |h|^{-2-2\alpha}\,(\Delta^h_{[\alpha]+1}F)\,(\Delta^h_{[\alpha]+1}U)\,\mathrm{d}x\,\mathrm{d}h\,\mathrm{d}t$$

for $F = \varrho\, g(T)$. Due to the corollary of Prop. 1 it suffices to estimate

$$\int_{R^5} |h|^{-2-2\alpha} (\Delta^h_{[\alpha]+2} F)(\Delta^h_{[\alpha]+2} U)\, dx\, dh\, dt < \tag{20}$$

$$< \frac{1}{\eta} \int_{R^5} |h|^{-2-(2\alpha-1+\varepsilon)} |\Delta^h_{[\alpha]+1} F|^2\, dx\, dh\, dt\, \frac{C_{2,[\alpha]+1}(\alpha)}{C_{2,[\alpha]+2}(\alpha)} +$$

$$+ \eta \int_{R^5 \times (0,r)} |h|^{-2-(2\alpha+1-\varepsilon)} |\Delta^h_{[\alpha]+2} U|^2\, dx\, dh\, dt\,,$$

$\varepsilon, \eta > 0$ arbitrarily small.

By the trace theorem and Prop. 1 the second term is bounded by

$$\int_{R^5+(0,r)} |h|^{-2-2\alpha} (\Delta^h_{[\alpha]+1}\, \nabla U, \Delta^h_{[\alpha]+1}\, \nabla U)_3\, dx\, dh\, dt\,,$$

cf. (15), hence it suffices to assume $F$ with bounded variation in time uniformly with respect to $x \in \partial\Omega$ and belonging to $L_2(0, \mathcal{T}; H^{27/14+\vartheta_2}(\partial\Omega))$, $\vartheta_2 > 0$ arbitrarily small. As $\frac{27}{14} + \vartheta_2 < 2$, it is sufficient to take the second differences of $F$ on the right hand side of (20). As the inequality

$$\int_0^\tau \int_{R^4} (\Delta^h_1\, \tilde{\nabla}\varrho\, g(T), \Delta^h_1\, \tilde{\nabla}\varrho\, g(T))_2\, |h|^{-2-13/7-2\vartheta_1}\, dx\, dh\, dt < +\infty \tag{21}$$

must hold, we can use (12) for $g(T)$ instead of $g(u)$ and in the end we arrive at the estimate like (16) with the exponent $-\frac{13}{14} - \vartheta_1$ at $|h|$ in the second term, where $u$ is replaced by $T$. To obtain its finiteness, we use Prop. 2 as follows

$$\begin{cases} 2\left(\frac{1}{2} - \frac{1}{2p}\right) + \frac{2}{\ell}\left(\frac{1}{2} - \frac{1}{2p}\right) + \frac{1}{\ell} < 1 \Rightarrow \frac{2}{\ell+1} < \frac{1}{p} \\ 2\frac{1}{2p} + \frac{2}{\ell}\frac{1}{2p} + \frac{1}{\ell}\frac{13}{14} < 1 \Rightarrow \ell > \frac{41}{14}, \end{cases} \tag{22}$$

where $\ell$ is the necessary generalized spatial regularity of $T$. In this way we have proved the following theorem.

**Theorem 1.** *Let $\Omega$ be a domain with a $C_{7/2}$-smooth boundary. Let $g$ from (1) be $C_2$-smooth, nondecreasing on $R_+$, let $g(0) = 0$. Let $\beta \equiv \beta_0$ be a positive constant, $\sigma \in (0, \frac{1}{2})$ a constant, let $\gamma$ from (2) be $C_2$-smooth, $E$ from (3) a continuous function, both on $R_+$. Let the heating regime $T$ have a uniformly (for $x \in \partial\Omega$) bounded variation as a function of time, let, moreover, $T \in L_2(0, \mathcal{T}; H^{41/14+\vartheta_1}(\partial\Omega))$ for some $\vartheta_1 > 0$. Then the corresponding stress tensor belongs to $C_0(\bar{Q}, R^9)$.*

Remark. 1. The same result could be reached for $T$ having classical first derivatives with respect to the space variables, if these derivatives are $\frac{13}{14} + \eta$-Hölder continuous uniformly with respect to $t \in \langle 0, \mathcal{T} \rangle$ for some $\eta > 0$.

2. For $T$ non-continuous in one of the space variables with discontinuities being of such type that $T \in \bigcap_{\varepsilon>0} L_2(0, \mathscr{T}; H^{1/2-\varepsilon}(\Gamma_c))$ it is still possible to prove $u \in \bigcap_{\varepsilon>0} C_0 \,.\, (0, \mathscr{T}; H^{6/5-\varepsilon}(\Omega))$. For $g(T) \in \bigcap_{\varepsilon>0} H^{1/2-\varepsilon, 1/2-\varepsilon, 9/2+\eta}(S)$ for some $\eta > 0$ and completely linear situation ($g$ is linear) it is possible to prove via the described localization and imbedding method that $u \in C_0(\bar{Q})$, $\tau \in C_0(\bar{Q}; R^9)$. Of course we need the boundary to be smoother along the variable in which $T$ behaves "smoothly" than was required in Theorem 1. For nonlinear $g$ the estimate corresponding to (10), (13), (16), (17) requires the fourth derivatives of composed functions and even a higher one in the estimate corresponding to (22), and also the use of the imbedding theorem is much more complicated. We avoid these technicalities.

The requirements concerning the behaviour of $T$ in the "bad" spatial direction can be even weakened, but then we must require much better behaviour in the second spatial direction. It is small hope to obtain the bounded and continuous stress tensor if $T$ is essentially discontinuous simultaneously in time and in both space variables.

## 2. THE INFLUENCE OF SUPPORTS

In this section we deal with a more general concept of a system (1), (2) for the two-dimensional model, as we shall study the influence of supports of the heated body. We suppose $\partial\Omega \equiv \Gamma = \Gamma_1 \cup \Gamma_2$, where $\Gamma_1$ is the nonsupported part of $\Gamma$ (with the boundary condition (2)) and $\Gamma_2$ is the supported one. For the sake of simplicity we suppose that $\Gamma_2$ is the union of a finite set of simple, bounded and smooth curves. $\bar{\Gamma}_1 \cup \bar{\Gamma}_2 \equiv M_0$ will be then a finite set. $\Gamma$ will be smooth with the exception of $M_0$ and an at most finite set $M_1$, where the nonsmoothnesses have the convex character (cf. [5] Sec. 4 or the text before Theorem 3 below). In our situation it is necessary to suppose (from the point of view of applications) that $g$ from (1) depends also on the space variable. We assume $g(x, 0) = 0$, $x \in \Gamma$, $g(x, \cdot)$ is non-decreasing for each $x \in \Gamma$, $g$ is continuously differentable on $\bar{\Gamma}_i \times R^1$, $i = 1, 2$. At the points of $M_0$ $g(\cdot, y)$ can have different limits with respect to $\Gamma_1$ and $\Gamma_2$, $y \in R^1$, and need not be continuous there. The assumptions concerning $\beta$ - its strict positivity, positivity of its first derivative on $B_{2\bar{B}}(0) \subset R^1$ and analyticity on the ball $B_{2\bar{B}}(0) \subset \subset C$ for a constant $\bar{B} \geqq 1$ – and the continuous differentiability of $\gamma$ will be the same as in [5] Thms. 4 and 5. The heating regime $T$ will be again non-negative, nondecreasing in time for each $x \in \Gamma$, bounded by the constant $\bar{B}$ on $S = (0, \mathscr{T}) \times \Gamma$, having $T(0, \cdot) = 0$ on $\Gamma$ and left continuous at $t = \mathscr{T}$ for each $x \in \Gamma$. The suppositions concerning $\sigma$ and $E$ will be the same as in Sec. 1.

For $\alpha \in (0, \frac{1}{2})$ it is easy to show under the above mentioned suppositions that for $T \in L_2(0, \mathscr{T}; H^\alpha(\Gamma))$, we have $g(\cdot, T(\cdot)) \in L_2(0, \mathscr{T}; H^\alpha(\Gamma))$. The proof of the following theorem is an easy modification of the proof of Theorem 4 of [5] and the regularity results of [2], [3] concerning the elliptic equations.

**Theorem 2.** *Under all the above mentioned suppositions concerning $\Omega$, $\mathscr{g}$, $\beta$, $T$ let $T \in L_2(0, \mathscr{T}; H^\alpha(\Omega))$, $\alpha \in (0, \frac{1}{2})$. Then there is a unique weak solution of (1) belonging to $\bigcap_{\varepsilon>0} C_0(0, \mathscr{T}; H^{1+\tilde{\alpha}-\varepsilon}(\Omega))$ together with $\gamma(u)$, where $\tilde{\alpha} = \tilde{\alpha}(\alpha) = (1 + 2\alpha)^2 :$ $: (14 + 12\alpha)$.*

Thus in this case the regularity of the stresses depends merely on the solution of the Lamé system, because the temperature is sufficiently regular. We need to prove the regularity only in the neighbourhood of the points of $M_0$. On $\Gamma_2$ we consider the following different type of boundary value conditions

$$(23) \qquad v_\nu = 0\,, \quad T_t \equiv (\tau_{ij}\nu_j) - (\tau_{ij}\nu_i\nu_j)\,\nu = 0\,;$$

$$(24) \qquad v = 0\,.$$

The first case corresponds to "zero friction" the second case to "friction equal to infinity".

First we will study the regularity problem for $\Omega$ being an angle, i.e. in polar coordinates

$$(25) \qquad \Omega \equiv V:= \{[r, \omega];\, r \in R^1_+,\, \omega \in (0, \omega_0)\}\,, \quad \text{where} \quad \omega_0 \in (0, 2\pi).$$

We suppose $R^1_+ \times \{0\} = \Gamma_2$, $R^1_+ \times \{\omega_0\} = \Gamma_1$ and the input function $\gamma(u)$ in (2) to be in some $C_0(0, \mathscr{T}; H^{1+\tilde{\alpha}}(\Omega))$ for $\tilde{\alpha} \in (0, \frac{1}{2})$. First we treat the case of "zero friction". We use the polar coordinates, transform $r = e^{\ell}$ and apply the Fourier transformation with respect to $\ell$. Introducing the notation $\mathscr{s} = 3 - 4\sigma$, $\tilde{v}$ the Fourier transform of $v$, $[v_r, v_\omega]$ its polar components, i.e. $v_r = v_1 \cos\omega + v_2 \sin\omega$, $v_\omega =$ $= v_2 \cos\omega - v_1 \sin\omega$, $\lambda$ the dual variable in the Fourier sense, $n = \mathrm{i}\lambda$ and $q_n =$ $= \widetilde{\gamma(u)}\,(\mathrm{i}(1 - n), \cdot)$, we obtain the system

$$(26) \qquad \left.\begin{aligned} &(\mathscr{s} - 1)\,\tilde{v}''_r + (\mathscr{s} + 1)\,(n^2 - 1)\,\tilde{v}_r + 2(n - \mathscr{s})\,\tilde{v}'_\omega = (7 - \mathscr{s})\,(n - 1)\,q_n \\ &(\mathscr{s} + 1)\,\tilde{v}''_\omega + (\mathscr{s} - 1)\,(n^2 - 1)\,\tilde{v}_\omega + 2(n + \mathscr{s})\,\tilde{v}'_r = (7 - \mathscr{s})\,q'_n \end{aligned}\right\},$$

$$\omega \in (0, \omega_0)$$

$$(\mathscr{s} - 1)\,(\tilde{v}'_r + (n - 1)\,\tilde{v}_\omega) = 0\,, \quad \omega = 0, \omega_0\,,$$

$$(\mathscr{s} + 1)\,(\tilde{v}'_\omega + \tilde{v}_r) + (3 - \mathscr{s})\,n\tilde{v}_r = (7 - \mathscr{s})\,q_n\,, \quad \omega = \omega_0\,,$$

$$v_\omega = 0\,, \quad \omega = 0\,.$$

The homogeneous system corresponding to (26) has nontrivial solutions for every $n_0$ solving the equation

$$(27) \qquad n_0 \sin(2\omega_0) = -\sin(2n_0\omega_0)\,,$$

and in the Cartesian components the solution has the form

$$(28)\qquad v_{n_0}(\omega) = [(\cos(2n_0\omega_0) + n_0\cos(2\omega_0) + \sigma)\cos(n_0\omega) - \\ - n_0\cos((n_0-2)\,\omega)\,, \\ (\sigma - \cos(2n_0\omega_0) - n_0\cos(2\omega_0))\sin(n_0\omega) + n_0\sin((n_0-2)\,\omega)]\,,$$

The correspondence between (27), (28) and (58), (61) of [5] is a consequence of the following facts: Let $V_1 \equiv \{[r, \omega];\ r \in R^1_+,\ |\omega| < \omega_0\}$ and let $\gamma(u)$ be a symmetric input function of (2) on $V_1$ (i.e. even in the variable $x_2$), $v(\gamma(u))$ the symmetric solution of (2) on $V_1$, i.e.

$$(29)\qquad v_1(\gamma(u))\,(x_1, -x_2) = v_1(\gamma(u))\,(x_1, x_2), \\ v_2(\gamma(u))\,(x_1, -x_2) = -v_2(\gamma(u))\,(x_1, x_2),\ [x_1, x_2] \in V_1\,.$$

Then $\tau_{ii}(v(\gamma(u)))$ is even in $x_2$ on $V_1$, $i = 1, 2$, $\tau_{12}(v(\gamma(u)))$ is odd in $x_2$ there. Therefore such $v(\gamma(u))$ solves our problem on $V$ for the given $\gamma(u)$. Conversely, if we extend $\gamma(u)$ and the solution $v(\gamma(u))$ of our problem (on $V$) symmetrically in the described manner onto $V_1$, then $v(\gamma(u))$ solves (2) on $V_1$ (if $v(\gamma(u))$ is sufficiently regular up to $\Gamma_2$).

Expressed in the Cartesian components, the solution of the problem (26) has the form

$$\widetilde{v_1(\gamma(u))}\,(-in, \omega) = P_r(n, \omega) + (A_1 + A_2\sigma)\cos(n\omega) - \\ - A_2 n\cos((n-2)\,\omega)\,,$$

$$\widetilde{v_2(\gamma(u))}\,(-in, \omega) = P_\omega(n, \omega) + (-A_1 + A_2\sigma)\sin(n\omega) + \\ + A_2 n\sin((n-2)\,\omega)\,,$$

$$(30)\qquad P_1(n, \omega) = \frac{1+\sigma}{1-\sigma}\int_0^\omega \sin(n\omega - (n+1)\,\xi)\,q_n(\xi)\,d\xi\,,$$

$$P_2(n, \omega) = \frac{1+\sigma}{1-\sigma}\int_0^\omega \cos(n\omega - (n+1)\,\xi)\,q_n(\xi)\,d\xi\,,$$

$$A_1 \equiv A_1(n) = \\ = \frac{1+\sigma}{1-\sigma}\int_0^{\omega_0} \frac{n\cos(2\omega_0 - (n+1)\,\xi) + \cos(2n\omega_0 - (n+1)\,\xi)}{\sin(2n\omega_0) + n\sin(2\omega_0)}\,. \\ .\, q_n(\xi)\,d\xi\,,$$

$$A_2 \equiv A_2(n) = \frac{1+\sigma}{1-\sigma}\int_0^{\omega_0} \frac{\cos((n+1)\,\xi)\,q_n(\xi)\,d\xi}{\sin(2n\omega_0) + n\sin(2\omega_0)}\,.$$

Considering a more general setting of the problem (26), i.e. a general right hand side $2\tilde{F} = [2\tilde{F}_r, \tilde{F}_\omega]$ with the argument $[i(1-n), \omega]$) in the equation and general right hand side $[2\tilde{K}^0_r(i(1-n)), 2\tilde{K}_r(i(1-n)), 2\tilde{K}_\omega(i(1-n)), \tilde{v}^0_\omega(-in)]$ in the boundary condition, the solution in the polar components attains the form

$$
\begin{aligned}
(31) \qquad \tilde{v}_r(-in, \omega) &= P_r(n, \omega) + A_1 \cos((n+1)\,\omega) + \\
&+ A_2(\mathscr{s} - n) \cos((n-1)\,\omega) + A_3 \sin((n+1)\,\omega) + \\
&+ A_4(n - \mathscr{s}) \sin((n-1)\,\omega)\,, \\
\tilde{v}_\omega(-in, \omega) &= P_\omega(n, \omega) - A_1 \sin((n+1)\,\omega) + \\
&+ A_2(\mathscr{s} + n) \sin((n+1)\,\omega) + \\
&+ A_3 \cos((n+1)\,\omega) + A_4(\mathscr{s} + n) \cos((n-1)\,\omega)\,,
\end{aligned}
$$

with $P$ and $\mathscr{o}$ defined as follows

$$
\begin{aligned}
P_r(n, \omega) &= \frac{1}{n(\mathscr{s}^2 - 1)} \int_0^\omega \tilde{F}_r(\mathrm{i}(1-n), \zeta)\,[(n + \mathscr{s})(\sin((n+1)(\omega - \zeta)) + \\
&+ (\mathscr{s} - n) \sin((n-1)(\omega - \zeta))] + \tilde{F}_\omega(\mathrm{i}(1-n), \zeta)\,. \\
&.\,[(n - \mathscr{s})(\cos((n+1)(\omega - \zeta)) - \cos((n-1)(\omega - \zeta)))]\,\mathrm{d}\zeta\,,
\end{aligned}
$$

$$
\begin{aligned}
(32) \qquad P_\omega(n, \omega) &= \frac{1}{n(\mathscr{s}^2 - 1)} \int_0^\omega \tilde{F}_r(\mathrm{i}(1-n), \zeta)\,[(n + \mathscr{s})(\cos((n+1)(\omega - \zeta)) - \\
&- \cos((n-1)(\omega - \zeta)))] + \tilde{F}_\omega(\mathrm{i}(1-n), \zeta)\,. \\
&.\,[(\mathscr{s} - n) \sin((n+1)(\omega - \zeta)) + (\mathscr{s} + n) \sin((n-1)(\omega - \zeta))]\,\mathrm{d}\zeta\,, \\
\mathscr{o}_r &\equiv \mathscr{o}_r(n, \omega_0) = \\
&= \frac{1}{\mathscr{s} + 1} \int_0^{\omega_0} \tilde{F}_r(i(1-n), \zeta)\,[(n + \mathscr{s}) \cos((n+1)(\omega - \zeta)) - \\
&- (n-1) \cos((n-1)(\omega - \zeta))] + \\
&+ \tilde{F}_\omega(\mathrm{i}(1-n), \zeta)\,[(\mathscr{s} - n) \sin((n+1)(\omega - \zeta)) + \\
&+ (n-1) \sin((n-1)(\omega - \zeta))]\,\mathrm{d}\zeta\,, \\
\mathscr{o}_\omega &\equiv \mathscr{o}_\omega(n, \omega_0) = \\
&= \frac{1}{\mathscr{s} + 1} \int_0^{\omega_0} \tilde{F}_r(\mathrm{i}(1-n), \zeta)\,[-(n + \mathscr{s}) \sin((n+1)(\omega - \zeta)) + \\
&+ (n+1) \sin((n-1)(\omega - \zeta))] + \\
&+ \tilde{F}_\omega(\mathrm{i}(1-n), \zeta)\,[(\mathscr{s} - n) \cos((n+1)(\omega - \zeta)) + \\
&+ (n+1) \cos((n-1)(\omega - \zeta))]\,\mathrm{d}\zeta\,,
\end{aligned}
$$

and the coefficients given by

(33)
$$A_1 \equiv A_1(n) =$$
$$= \frac{(n+1)\, Z_1 \cos((n-1)\,\omega_0) - (n-1)\, Z_2 \sin((n-1)\omega_0)}{\sin(2n\omega_0) + n\sin(2\omega_0)} +$$
$$+ \frac{A_3(n\cos(2\omega_0) + \cos(2n\omega_0)) + A_4(n^2-1)}{\sin(2n\omega_0) + n\sin(2\omega_0)},$$
$$A_2 \equiv A_2(n) = \frac{Z_1\cos((n+1)\,\omega_0) - Z_2\sin((n+1)\,\omega_0)}{\sin(2n\omega_0) + n\sin(2\omega_0)} +$$
$$+ \frac{A_3 + A_4(n\cos(2\omega_0) - \cos(2n\omega_0))}{\sin(2n\omega_0) + n\sin(2\omega_0)},$$
$$A_3 \equiv A_3(n) = -\frac{1}{\mathscr{s}+1}\left((n-1)\,\tilde{v}^0_\omega(-in) - \frac{(n+\mathscr{s})}{n(\mathscr{s}-1)}\,\tilde{K}^0_r(\mathrm{i}(1-n))\right),$$
$$A_4 \equiv A_4(n) = -\frac{1}{\mathscr{s}+1}\left(\frac{\tilde{K}^0_r(\mathrm{i}(1-n))}{n(\mathscr{s}-1)} - \tilde{v}^0_\omega(-\mathrm{i}n)\right),$$
$$Z_1 \equiv Z_1(n) = \frac{1}{n(\mathscr{s}-1)}\,(\mathscr{o}_r(n,\omega_0) - \tilde{K}_r(\mathrm{i}(1-n))),$$
$$Z_2 \equiv Z_2(n) = \frac{1}{n(\mathscr{s}-1)}\,(\mathscr{o}_\omega(n,\omega_0) - \tilde{K}_\omega(\mathrm{i}(1-n))).$$

Of course, for $n$ solving (27) the Green operator, whose Fourier transform is expressed in (30) or (31), has a pole. There are only single or double poles, the double poles must fulfil the following additional condition to (27)

(34)
$$\cos(2n_0\omega_0) = -\frac{\sin(2\omega_0)}{2\omega_0}$$

which together with (27) implies that $n_0^2 = 1/(\sin^2(2\omega_0)) - 1/(4\omega_0^2)$ for $\omega_0 \neq (k/2)\,\pi$, $k$ an integer. For a given $\omega_0$ there is at most one pair of double poles. For $\omega_0 = (k/2)\,\pi$ with $k$ being an integer, there are no double poles. It is possible to calculate that the singular function (a function solving the homogeneous problem) corresponding to a pole has the form $r^{n_0}\, v_{n_0}(\omega)$ with $v_{n_0}$ from (28). For $n_0 = 0$ such a singular function is $[1, 0]$, i.e. the shift in the tangential direction. For the single poles these are the unique singular functions. For a double pole the appropriate second singular function has the form $r^{n_0} \ln r \,.\, \mathscr{w}_{n_0}(\omega)$, where (in the Cartesian components)

(35)
$$\mathscr{w}_{n_0}(\omega) = [(\mathscr{C} + \mathscr{s})\,\omega\sin(n_0\omega) - n_0\omega\sin((n_0-2)\,\omega) +$$
$$+ \cos((n_0-2)\,\omega) + \mathscr{D}\cos(n_0\omega),$$
$$(\mathscr{C} - \mathscr{s})\,\omega\cos(n_0\omega) - n_0\omega\cos((n_0-2)\,\omega) - \sin((n_0-2)\,\omega) -$$
$$- \mathscr{D}\sin(n_0\omega)],$$
$$\mathscr{C} = \cos(2n_0\omega_0) + n_0\cos(2\omega_0), \quad \mathscr{D} = 2\omega_0\sin(2n_0\omega_0) - \cos(2\omega_0).$$

As usual, $w_{n_0}$ satisfies the system (26) with the right hand side $2[n_0(\sigma + 1) v_{n_0,r} + v'_{n_0,\omega}, n_0(\sigma - 1) v_{n_0,\omega} + v'_{n_0,r}]$ and the right hand side of the boundary value condition $[\tilde{v}^0_\omega, \tilde{K}^0_r] = [0, 0]$ at $\omega = 0$ and $2[\tilde{K}_r, \tilde{K}_\omega] = [(\sigma - 1) v_{n_0,\omega}(\omega_0), (3 - \sigma) v_{n_0,r}(\omega_0]$ at $\omega = \omega_0$ (cf. [2], [7], [11]).

Using the weighted Sobolev spaces with fractional derivatives as in [5] Sec. 4, the technique based on the Cauchy residuum theorem as in [6], [7], [10] with the help of the proof that the weak solution of the problem is in the space $H^2_{1+\alpha_0}(V)$ for some small $\alpha_0 > 0$ (for definition of the weighted spaces we refer e.g. to [5], [7], the idea of the proof is in [6], [11]), we prove similarly to [5] the regularity, i.e.the boundedness and continuity of the stress for $\omega_0 < \pi/2$, because there is no pole with Re $n_0 \in (-1, 1 + \eta_0)$ for suitably small $\eta_0$ for such angles. Of course, we need to avoid the boundary value condition that are not in the appropriate weighted Sobolev space $H_0^{1/2+\varepsilon_0}(\Gamma_1)$ for some $\varepsilon_0 > 0$, but this can be done with the help of the same auxiliary function as in [5] Sec. 4 (cf. (64) there). For $\omega_0 = \pi/2$, $n_0 = 1$ is a pole and the corresponding singular function is

$$(36) \qquad [x_1, x_2] \mapsto [-\sigma x_1, (1 - \sigma) x_2] .$$

The stress tensor corresponding to it is $\tau_{11} = \tau_{12} = \tau_{21} \equiv 0$, $\tau_{22} \equiv 1$ on $V$, i.e. it is continuous and bounded. All the poles are integers. Thus the weak solution for $\omega_0 = \pi/2$ is also regular, the corresponding stress is continuous and bounded on $V$. For $\omega_0 > \pi/2$ there is a very small hope of obtaining the weak solution having the stress bounded and continuous at the origin (cf. [7] Thms. 7.3 and 12.5 – there is a small hope of obtaining zero coefficients at the singular functions in the corresponding expansion for each $t \in (0, \mathcal{T})$).

The case with "friction equal to infinity" can be studied in the same way as the preceding case. The Fourier transform of the solution of the Lamé system must satisfy the equation (26) with the boundary value condition $v(0) = 0$ and for $\omega = \omega_0$ the condition given in (26). The equation for poles has the form

$$(37) \qquad \mathscr{I}(n_0, \omega_0, \sigma) = n_0^2 \sin^2 \omega_0 + \sigma \sin^2 n_0\omega_0 - \left(\frac{\sigma + 1}{2}\right)^2 = 0 ,$$

the corresponding singular solution of the modified system (26) has the form (in the Cartesian components)

$$(38) \qquad v_{n_0}(\omega) = [\mathscr{I}_1, \mathscr{I}_2]\, n_0(\sin((n_0 - 2)\,\omega) - \sin(n_0\omega)) +$$

$$+ [\mathscr{I}_2, -\mathscr{I}_1]\, n_0(\cos(n_0\omega) - \cos((n_0 - 2)\,\omega)) +$$

$$+ 2\sigma[-\mathscr{I}_1, \mathscr{I}_2] \sin(n_0\omega), \quad \mathscr{I}_1 = 2n_0 \sin^2\omega_0 - \cos(2n_0\omega) - \sigma ,$$

$$\mathscr{I}_2 = n_0 \sin(2\omega_0) - \sin(2n_0\omega_0) .$$

The additional condition for the double poles to (35) is the following

$$(39) \qquad n_0 \sin^2 \omega_0 + \sigma\omega_0 \sin(n_0\omega_0)\cos(n_0\omega_0) = 0\,, \quad \text{hence}$$

$$n_0 = \pm \frac{1}{\sin \omega_0} \sqrt{\left(\frac{\sigma^2 + 1}{4} - \frac{\mathcal{S}}{2} \pm \frac{1}{2}\sqrt{((\mathcal{S} - \sigma^2)(\mathcal{S} - 1))}\right)},$$

$$\mathcal{S} = \frac{\sin^2 \omega_0}{\omega_0^2}\,, \quad \text{for} \quad \omega_0 \neq \pi\,.$$

For $\omega_0 \neq \pi$ there can be at most 4 real double poles, for $\omega_0 = \pi$ there is no double pole. The first singular function corresponding to the pole $n_0$ has the form $r^{n_0}\, v_{n_0}(\omega)$, $v_{n_0}$ from (38). For the double poles there is a second singular function having the form $r^{n_0} \ln r\, w_{n_0}(\omega)$, where $w_{n_0}$ is the solution of the system (26) with the right hand side $2[n_0(\sigma + 1)\, v_{n_0,r} + v'_{n_0,\omega},\ n_0(\sigma - 1)\, v_{n_0,\omega} + v'_{n_0,r}]$ and the boundary condition $w_{n_0}(0) = 0$, while for $\omega = \omega_0$ the boundary condition is as in (26) with the right hand side $[(\sigma - 1)\, v_{n_0,\omega}(\omega_0),\ (3 - \sigma)\, v_{n_0,r}(\omega_0)]$, where $v_{n_0}$ is given by (38). We avoid the calculation of $w_{n_0}$ for this case.

The Fourier transform of the solution of the Lamé system with the above described boundary condition can be expressed in the Cartesian components as

$$(40) \qquad \tilde{v}_1(n, \omega) = \frac{1 + \sigma}{1 - \sigma}\Bigg(\int_0^\omega \sin(n\omega - (n + 1)\tau)\, q_n(\tau)\, d\tau +$$
$$+ D_1(n \sin((n - 2)\omega) - (n + 2\sigma)\sin(n\omega)) + D_2 n(\cos(n\omega) -$$
$$- \cos((n - 2)\omega))\Bigg),$$

$$\tilde{v}_2(n, \omega) = \frac{1 + \sigma}{1 - \sigma}\Bigg(\int_0^\omega \cos(n\omega - (n + 1)\tau)\, q_n(\tau)\, d\tau +$$
$$+ D_1(n \cos((n - 2)\omega) -$$
$$- \cos(n\omega)) + D_2(n \sin((n - 2)\omega) + (2\sigma - n)\sin(n\omega))\Bigg),$$

$$D_1 \equiv D_1(n) = (4\mathcal{J}(n, \omega_0, \sigma))^{-1}\,((2n \sin^2 \omega_0 - \sigma - \cos(2n\omega_0))\,.$$
$$. \textstyle\int_0^{\omega_0} \cos((n + 1)\tau)\, q_n(\tau)\, d\tau - (n \sin(2\omega_0) + \sin(2n\omega_0))\,.$$
$$. \textstyle\int_0^{\omega_0} \sin((n + 1)\tau)\, q_n(\tau)\, d\tau)\,,$$
$$D_2 \equiv D_2(n) = (4\mathcal{J}(n, \omega_0, \sigma))^{-1}\,((2n \sin^2 \omega_0 + \sigma + \cos(2n\omega_0))\,.$$
$$. \textstyle\int_0^{\omega_0} \sin((n + 1)\tau)\, q_n(\tau)\, d\tau + (n \sin(2\omega_0) - \sin(2n\omega_0))\,.$$
$$. \textstyle\int_0^{\omega_0} \cos((n + 1)\tau)\, q_n(\tau)\, d\tau)\,, \quad \mathcal{J}(n, \omega_0, \sigma) \quad \text{from (37).}$$

We remark that for the general case with the right hand side $2\tilde{F}(i(1 - n), \omega) = 2[\tilde{F}_r, \tilde{F}_\omega](i(1 - n), \omega)$ of the Lamé system, the right hand side of the boundary value condition $\tilde{v}^0(-in) = [v_r^0, \tilde{v}_\omega^0](-in)$ at $\omega = 0$ and $2\tilde{K}(i(1 - n)) = 2[\tilde{K}_r, \tilde{K}_\omega]\,.\,(i(1 - n))$ at $\omega = \omega_0$, the Fourier transform of the solution in the polar components can be expressed as in (31) with $P$ and $\varrho$ from (32) but with the coefficients defined in (41), where $\mathscr{I}(n, \omega_0, \sigma)$ is from (37) and $Z_1 \equiv Z_1(n) = (1/n(\sigma - 1)) (\tilde{K}_r(i(1 - n)) - \varrho_r(n, \omega_0))$, $Z_2 \equiv Z_2(n) = (1/n(\sigma - 1)) (K_\omega(i(1 - n)) - \varrho_\omega(n, \omega_0))$:

$$A_1 = (4\mathscr{I}(n, \omega_0, \sigma))^{-1} (Z_1((n - \sigma)(n + 1) \sin((n - 1)\omega_0) + (\sigma^2 - n^2) \sin((n + 1)\omega_0)) + Z_2((n - \sigma)(n - 1) \cos((n - 1) . \omega_0) + (\sigma^2 - n^2) \cos((n + 1)\omega_0)) + \tilde{v}_r^0(-in)(n^2 - 1 - (n + \sigma) \cos(2n\omega_0) + n \cos(2\omega_0))) + \tilde{v}_\omega^0(-in)(n - \sigma)(n \sin(2\omega_0) - \sin(2n\omega_0)),$$

(41)
$$A_2 = (4\mathscr{I}(n, \omega_0, \sigma))^{-1} (Z_1((n + 1) \sin((n - 1)\omega_0) - (n + \sigma) \sin((n + 1)\omega_0)) + Z_2((n - 1) \cos((n - 1)\omega_0) - (n + \sigma) \cos((n + 1)\omega_0)) - v_r^0(-in)((n + \sigma) + \cos(2n\omega_0) - n \cos(2\omega_0)) + v_\omega^0(-in)(n \sin(2\omega_0) - \sin(2n\omega_0)),$$

$$A_3 = (4\mathscr{I}(n, \omega_0, \sigma))^{-1} (Z_1((n^2 - \sigma^2) \cos((n + 1)\omega_0) - (n + \sigma)(n + 1) \cos((n - 1)\omega_0)) + Z_2((n + \sigma)(n - 1) . \sin((n - 1)\omega_0) + (\sigma^2 - n^2) \sin((n + 1)\omega_0)) - v_r^0(-in)(n + \sigma)(n \sin(2\omega_0) + \sin(2n\omega_0)) + \tilde{v}_\omega^0(-in)(n^2 - 1 + (\sigma - n)(n \cos(2\omega_0) - \cos(2n\omega_0))),$$

$$A_4 = (4\mathscr{I}(n, \omega_0, \sigma))^{-1} (Z_1((n + 1) \cos((n - 1)\omega_0) - (n - \sigma) \cos((n + 1)\omega_0)) + Z_2((1 - n) \sin((n - 1)\omega_0) + (n - \sigma) \sin((n + 1)\omega_0)) + \tilde{v}_r^0(-in)(n \sin(2\omega_0) + \sin(2n\omega_0)) - \tilde{v}_\omega^0(-in)((n - \sigma) - n \cos(2\omega_0) - \cos(2n\omega_0))),$$

$$A_j \equiv A_j(n), \quad j = 1, 2, 3, 4.$$

As in the preceding case for the regularity of the weak solution of the problem, the existence of the eigenvalues $n_0$ with $\operatorname{Re} n_0$ in $\langle 0, 1 \rangle$ is essential. The equality (37) can be written separately for the real and imaginary parts, if we denote $n_1 = \operatorname{Re} n_0$, $n_2 = \operatorname{Im} n_0$:

(42)
$$(n_2^2 - n_1^2) \sin^2 \omega_0 + \sigma(\operatorname{sh}^2(n_2\omega_0) \cos^2(n_1\omega_0) - \operatorname{ch}^2(n_2\omega_0) \sin^2(n_1\omega_0)) + \left(\frac{\sigma + 1}{2}\right)^2 = 0,$$

$$\frac{\sin^2\omega_0}{\omega_0^2} = -\sigma \frac{\sin(2n_1\omega_0)}{2n_1\omega_0} \frac{\operatorname{sh}(2n_2\omega_0)}{2n_2\omega_0}, \quad n_1, n_2 \neq 0$$

(for $n_1 n_2 = 0$ only the first equation remains).

Let $\omega_0 \in (0, \pi/2)$. Then for $n_2 \neq 0$ the second equation of (42) cannot have a solution with $|n_1| \leqq 1$. If $n_2 = 0$, then the first equation of (42) has the form $\varphi_0(n_1) \equiv$ $\equiv n_1^2 \sin^2 \omega_0 + \sigma \sin^2 (n_1 \omega_0) = ((\sigma + 1)/2)^2$. Clearly for $n_1 \in \langle 0, 1 \rangle$ $\varphi_0$ is nondecreasing in $n_1$, the inequality $\varphi_0(1) = (\sigma + 1) \sin^2 \omega_0 < ((\sigma + 1)/2)^2$ is valid only if $\omega_0 < \arcsin (\sqrt{(1 - \sigma)})$. For $\omega_0 > \pi/2$ it is possible to calculate that there is a pole $n_0$ with $n_1 = \operatorname{Re} n_0 \in (0, 1)$. We remark that for $\omega_0 = \pi$ such $n_0$ has the form $\frac{1}{2} \pm (i/\pi) \operatorname{Argch} ((2 - 2\sigma)/\sqrt{(3 - 4\sigma)})$. To be able to apply the technique of the weighted Sobolev spaces with functional derivatives based on results of [6], [7], [10], [5], ... as in the preceding case, we need to have the right hand side of the Neumann condition in $H_0^{1/2+\varepsilon_0}(V)$ for some $\varepsilon_0 > 0$. To achieve this we use the following auxiliary function $v_0: R^2 \to R^2$:

$$(43) \qquad v_0: [x_1, x_2] \mapsto \varrho(\sqrt{(x_1^2 + x_2^2)}) \frac{\gamma(u)(0,0)(1 + \sigma)}{1 - \sigma - \sin^2 \omega_0} x_2 \, .$$

$$. [-\sin 2\omega_0, \cos 2\omega_0] \, , \quad \omega_0 \neq \arcsin (\sqrt{(1 - \sigma)}) \, ,$$

where $\varrho \equiv 1$ on $\langle 0, \eta)$, $\varrho \equiv 0$ on $(2\eta, +\infty)$ for some $\eta > 0$, $\varrho$ sufficiently smooth on $\langle 0, +\infty)$. Then for the weak solution $v$ of our problem we have $v - v_0 = 0$ on $\Gamma_2$ and on $\Gamma_1$ $v - v_0$ satisfies the Neumann boundary condition being in $H_0^{1/2+\varepsilon_0}(\Gamma)$, $\varepsilon_0 \in \langle 0, \tilde{\alpha})$. Therefore for $\omega_0 < \arcsin (\sqrt{(1 - \sigma)})$ we can prove the regularity of the weak solution – the continuity and boundedness of the stresses. For $\omega_0 \geqq \arcsin (\sqrt{(1 - \sigma)})$ there is a very small hope of obtaining such a result.

Via the localization technique we can extend (as in Sec. 4 of [5]) the above results to the case of a bounded domain $\Omega$ with $C_{2+\varepsilon}$-smooth boundary, $\varepsilon > 0$, with the exception of the points of $M_0 = \bar{\Gamma}_1 \cap \bar{\Gamma}_2$ and a finite set $M_1 \subset \Gamma_1$, where $\Gamma_1$ can be nonsmooth, but $\Gamma \in C_{0,1}$. For the sake of simplicity we suppose that for each $x_0 \in M_0 \cup M_1$ there is a neighbourhood $\mathscr{V}_{x_0}$ of $x_0$ and an open angle $V_{x_0}$ with the vertex at $x_0$ such that $\mathscr{V}_{x_0} \cap \Omega = \mathscr{V}_{x_0} \cap V_{x_0}$. For $x_0 \in M_0$ we will call $V_{x_0}$ the contact angle. Thus we have proved the following theorem.

**Theorem 3.** *Under the assumptions of Theorem 2 let for the Lamé system the Neumann boundary condition and the condition (23) be prescribed on $\Gamma_1$ and on $\Gamma_2$, respectively, such that the above mentioned assumption concerning $\partial\Omega$, $M_0$, $M_1$ and the angles holds. Let all the contact angles be less than or equal to $\pi/2$ and let all angles corresponding to the points of $M_1$ be less than $\pi$. Then the stresses corresponding to the solution of the thermoelastic system (1), (2) are continuous and bounded on $\bar{Q}$. If the condition (24) is prescribed on $\Gamma_2$ instead of (23), all the contact angles are less than arcsin $(\sqrt{(1 - \sigma)})$ and the other assumptions remain valid, the assertion concerning the stresses remains true.*

Remark. 1. We are able to extend the results of Theorem 3 to the threedimensional model considered in Sec. 1 in the sense of the remark at the end of Section 1 to the case, when the boundary $\partial\Gamma_2$ of the contact part of $\delta\Omega$ is sufficiently smooth and the heating regime "along" $\partial \Gamma_2$ is sufficiently smooth, too. The requirements of Theorem 3 in this case concern the angles in the normal direction to $\partial \Gamma_2$. The same is true for nonsmoothnesses of $\Gamma_1$ having the form of edges. Of course. it is possible to suppose also an analogous nonsmoothness of $\Gamma_2$.

2. If the contact condition is described by the Signorini boundary value condition ($v_\nu \leqq 0$, $T_\nu \equiv \tau_{ij} \nu_i \nu_j \leqq 0$, $v_\nu T_\nu = 0$) without friction ($T_t = 0$ as in (23)), it seems that we obtain the same results as for the case (23) of the boundary value condition at least for the two-dimensional model. In fact, denoting the contact part of the boundary by $\Gamma_c$, supposing it to be a line segment (for the sake of simplicity) and assuming that the set $\partial \operatorname{supp} v_n \cap \Gamma_c$ is finite (for a function $f: M \to R^1$, $\operatorname{supp} f = \overline{\{x \in M; f(x) \neq 0\}}$, $\partial$ denotes the boundary), we are able to prove that in the neighbourhood of the points of $\partial \operatorname{supp} v_n$ the stress is bounded and continuous – the only singular function corresponding to that transition for $\omega_0 = \pi$ is of the type (36), the others do not fulfil all the Signorini requirements. At the transition points $\bar{\Gamma}_c \cap \bar{\Gamma}_1$ we are in the same situation as at the points of $M_0 = \bar{\Gamma}_2 \cap \bar{\Gamma}_1$ with the condition (23) on $\Gamma_2$.

## 3. CONCLUSION

In both parts of the paper we have studied the solution of the quasilinear noncoupled thermoelastic model mainly from the regularity point of view in the sense of the continuity and boundedness of the stresses. For technical practice the model seems to be satisfactory only if such a regularity of the solution is satisfied. As the singularities of the boundary itself or the qualitative transitions of the boundary value condition are more important for the possible local nonregularity of the stresses than the singularities of the heating regime, we summarize the results concerning the admissible angles for various situations in the following table, where

Table 1.

| 1. | 2. | 3. |
|---|---|---|
| Neumann/Neumann | (i) $\sin(n_0 \omega_0) = \pm n_0 \sin \omega_0$ | $\omega_0 \in (0, \pi\rangle$ |
| Dirichlet/Dirichlet | (ii) $\vartheta \sin(n_0 \omega_0) = \pm n_0 \sin \omega_0$ | $\omega_0 \in (0, \pi\rangle$ |
| Neumann/normal Dirichlet, tangential Neumann | (iii) $\sin(2 n_0 \omega_0) = -n_0 \sin(2\omega_0)$ | $\omega_0 \in (0, (\pi/2)\rangle$ |
| Neumann/Signorini | (i) or (iii), $n_0 \in R^1$ *) | $\omega_0 \in (0, (\pi/2)\rangle$ **) |
| Neumann/Dirichlet | (iv) $n_0^2 \sin^2 \omega_0 + \vartheta \sin(2n\omega_0) = ((\vartheta + 1)/2)^2$ | $\omega_0 \in (0, \arcsin(\sqrt{1 - \sigma}))$ |

*) The Signorini inequality must hold for the appropriate singular solution.

**) Under the suppositions of Remark. 2.

in the first column the type of the boundary singularity or the transition of the boundary condition is introduced, in the second the equation for the poles and in the third the admissible magnitude of the angle to ensure the regularity are indicated.

*References*


[1] *O. V. Běsov, V. P. Iljin, S. M. Nikol'skij:* Integral Transformations of Functions and Imbedding Theorems (in Russian). Nauka, Moskva 1975.

[2] *P. Grisvard:* Elliptic Problems in Nonsmooth Domains. Monographs and Studies in Math. 24, Pitman, Boston—London—Melbourne 1985.

[3] *P. Grisvard:* Problèmes aux limites dans les polygones. Mode d'emploi. EDF Bull. Direct. Etud. Rech. Ser. C - Math. Inform. (1986), 21—59.

[4] *J. Jarušek:* Contact problems with bounded friction. Coercive case. Czech. Math. J. 33 (108) (1983), 237—261.

[5] *J. Jarušek:* On the regularity of solutions of a thermoelastic system under noncontinuous heating regimes. Apl. Mat. 35 (1990) 6, 426—450.

[6] *V. A. Kondratěv:* Elliptic boundary value problems with conical or angular points (in Russian). Trudy Mosk. Mat. Obšč., Vol. 16 (1967), 209—292.

[7] *A. Kufner, A. M. Sändig:* Some Applications of Weighted Sobolev Spaces. Teubner-Texte Math, Vol. 100, Teubner V., Leipzig 1987.

[8] *O. A. Ladyženskaya, V. A. Solonnikov, N. N. Uraltseva:* Linear and Quasilinear Equations of Parabolic Type (in Russian), Nauka, Moskva 1967.

[9] *J. L. Lions, E. Magenes:* Problèmes aux limites non-homogènes et applications. Dunod, Paris 1968.

[10] *V. G. Maz'ja, B. A. Plamenĕvskij:* On the coefficients in asymptotics of solutions of elliptic boundary value problems in domains having conical points (in Russian), Math. Nachr.76 (1977), 29—60.

[11] *A. M. Sänding, U. Richter, R. Sänding:* The regularity of boundary value problems for the Lamé equation in polygonal domain. Rostock Math. Kolloq. 36 (1989), 21—50.

[12] *A. Visintin:* Sur le problème de Stefan avec flux non-linéaire Preprint No 230, Ist. Anal. Numer, C. N. R. Pavia, Pavia 1981.


Souhrn

## REGULARITA ŘEŠENÍ TERMOELASTICKÉHO SYSTÉMU S NESPOJITÝMI REŽIMY OHŘEVU. II. ČÁST


JIŘÍ JARUŠEK



Postačující podmínky pro spojitost a omezenost napětí na uzávěru časoprostorového válce se odvozují pro kvazilineární termoelastický systém na trojrozměrné omezené oblasti a pro zobecněný systém zahrnující vliv podpěr.



*Author's address:* RNDr. *Jiří Jarušek*, CSc., Institute of Information Theory and Automation, Czechoslovak Academy of Sciences, Pod vodárenskou věží 4, 182 08 Praha 8, Czechoslovakia.