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ON THE RATE OF APPROXIMATION IN THE RANDOM SUM CLT FOR DEPENDENT VARIABLES

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Summary. Capital "O" and lower-case "o" approximations of the expected value of a class of smooth functions ($f \in C^r(R)$) of the normalized random partial sums of dependent random variables by the expectation of the corresponding functions of Gaussian random variables are established. The same types of approximation are also obtained for dependent random vectors. This generalizes and improves previous results of the author (1980) and Rychlik and Szynal (1979).

Keywords: dependent random variables, random vectors, Random Sum CLT.
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1. INTRODUCTION

Let $\{X_n, n \geqq 1\}$ be a sequence of random variables defined on a probability space $(\Omega, \mathscr{F}, P)$. Denote by N a positive integer-valued random variable which has a distribution function dependent on a parameter $\lambda(\lambda > 0)$, i.e. $P[N = n] = p_n$, where $p_n = p_n(\lambda)$, $n = 1, 2, \ldots$ are functions of λ. We assume that, for every λ, the random variables $N, X_1, X_2, \ldots$ are independent. Let us put $S_n = \sum_1^n X_k$. Recently, several papers have appeared which are devoted to the study of the limit distribution of S_N. The first result of this type has been established by Robbins (1948). He gave, in the case of independent and identically distributed random variables X_n, $n = 1, 2, \ldots$ sufficient conditions for the relation

$$\lim_{\lambda \to \infty} P[S_N - \mathsf{E}S_N < x \sqrt{\operatorname{Var} S_N}] = \Phi(x). \tag{1.0}$$

to hold, where $\Phi(x)$ is the standard normal distribution function. Equivalent to (1.0) is

$$\lim_{\lambda \to \infty} \int_R f(x)\, dF_\lambda(x) = \int_R f(x)\, d\Phi(x) \tag{1.1}$$

for each $f \in C_B^r(R)$ where $F_\lambda(x) = P[S_N - \mathsf{E}S_N < x\sqrt{\mathsf{Var}\, S_N}]$ and $C_B^r(R) = \{f \in C_B(R): f^{(j)} \in C_B(R), 1 \leqq j \leqq r\}$, $\|f\| = \sup_x |f(x)|$, and $C_B(R)$ denotes the class of bounded uniformly continuous functions defined on R. By the same method some generalizations of Robbins' result have been obtained by Mamatov and Nematov (1971), Rychlik and Szynal (1973), and Sirazhdinov and Orazov (1966). The dependent case was treated by Rychlik and Rychlik (1980).

A rate of convergence of (1.1) was established by Rychlik and Szynal (1979). We have generalized Rychlik's and Szynal's results to the dependent case and to random vectors. Our results are generalizations of the result given by Basu (1980) and Butzer et al. (1975). The novelty of the present paper lies in the fact that it uses elementary techniques.

Throughout the paper, the relations of equality or inequality stated between random variables or random vectors are to be understood to hold only almost surely. (I($\cdot$) denotes the indicator of the set within the brackets. Let there exist a sequence of σ-algebras $\mathscr{F}_0 \subset \mathscr{F}_1 \subset \ldots \subset \mathscr{F}_n \subset \mathscr{F}$ and let X_n be $\mathscr{F}_n$ measurable for all $n \geqq 1$. We denote the conditional expectation with respect to $\mathscr{F}_{k-1}$ by E_{k-1} and the conditional distribution function with respect to $\mathscr{F}_{k-1}$ by $F_{k-1}^*(\cdot)$. Throughout the paper the random variable N and the random sequence $\{X_n\}$ are *independent* and the X_n's form a sequence of *martingale differences* with respect to the sequence of σ-algebras $\mathscr{F}_n$.

2. "CAPITAL O" APPROXIMATION

Assume

$$\mathsf{E}(X_k \mid \mathscr{F}_{k-1}) = 0\,, \quad k = 1, 2, \ldots, n\,. \tag{2.1}$$

Denote

$$\sigma_k^2 = E_{k-1}(X_k^2)\,, \quad M^2 = \sum_1^N \sigma_k^2\,, \quad \beta_{r,k} = \mathsf{E}|X_k|^r\,,$$

$$\gamma_r = \int |x|^r \,\mathrm{d}\Phi(x)\,, \quad T_N^r = \sum_{k=1}^N \beta_{r,k}\,, \quad V_n^2 = \sum_1^n \sigma_k^2\,.$$

Let

$$v_k(i) = \int x^i \,\mathrm{d}[F_k^*(xV_n) - \Phi(xV_n/\sigma_k)] = 0\,, \tag{2.2}$$

$w(f; \varepsilon) = \sup_{|h| \leqq \varepsilon} |f(x+h) - f(x)|$, $\varepsilon > 0$. If $w(f, \varepsilon) \leqq k\varepsilon^\alpha$, where $0 < \alpha \leqq 1$ and k is a positive constant, then we write $f \in \mathrm{Lip}\,(\alpha, k)$.

Theorem 2.1. *Let $\{X_n\}$ be a sequence of random variables satisfying* (2.1) *and* (2.2) *for $2 \leqq i < r$ and $\beta_{r,k} < \infty$ $(1 \leqq k \leqq n)$ for some positive integer $r \geqq 3$.*

Then for any $f \in C_B^{r-1}(R)$,

$$|\mathsf{E}[f(S_N/M)] - \int_R f(x)\, \mathrm{d}\Phi(x)|$$

$$\leqq 2\gamma_r k\, \mathsf{E}\{M^{1-r} w(f^{(r-1)};\, M^{-1})\,(T_N^{r-1} + T_N^r)\}/(r-1)!$$

If, in addition, $f^{(r-1)} \in \mathrm{Lip}\,(\alpha, k)$, $0 \leqq \alpha < 1$, *then*

$$|\mathsf{E}[f(S_N/M)] - \int_R f(x)\, \mathrm{d}\Phi(x)|$$

$$\leqq 4K\gamma_r \operatorname{Max}\{\mathsf{E}(T_N^r/M^{r+\alpha-1})\,;\ \mathsf{E}(T_N^{r-1}/M^{r+\alpha-1})\}/(r-1)!\,.$$

Proof. As in Basu [1980], denote

$$Z_k = V_n^{-1}(X_1 + \ldots + X_{k-1}) + (\sigma_{k-1}Y_{k-1} + \ldots + \sigma_n Y_n)\, V_n^{-1} \quad \text{for}$$

$$k = 1, 2, \ldots, n\,,$$

where $\{Y_k\}$ are i.i.d. $N(0, 1)$ r.vs, independent of N and $\{X_k\}$. Then

$$|\mathsf{E}[f(S_N/M)] - \int_R f(x)\, \mathrm{d}\Phi(x)| =$$

$$= |\sum_{n=1}^{\infty} p_n\{\mathsf{E}|f(S_n/\sqrt{\textstyle\sum_1^n \sigma_k^2}) - \int_R f(x)\, \mathrm{d}\Phi(x)|\} \leqq$$

$$\leqq \sum_{n=1}^{\infty} p_n \sum_{k=1}^{n} \mathsf{E}\,|\,\mathsf{E}^*[f(Z_k + X_k V_n^{-1}) - f(Z_k + \sigma_k Y_k V_n^{-1})]|\,.$$

The rest of the proof follows from Theorem 1 of Basu [1980].
Let $\sigma_1^2 = \sigma^2 = E(X_1^2 \mid \mathscr{F}_0) = \mathsf{E}X_1^2$, $\mathscr{F}_0 = \{\Phi, \Omega\}$.

Corollary 2.1. *Let* $\{X_n, n \geqq 1\}$ *be a stationary ergodic sequence of random variables satisfying* (2.1), (2,2), *and* $\beta_{1,r} = \mathsf{E}|X_1|^r < \infty$ *for* $r \geqq 3$. *Then for any* $f \in C_B^{r-1}(B)$,

$$|\mathsf{E}f(S_N/\sqrt{N}\sigma) - \int_R f(x)\, \mathrm{d}\Phi(x)| \leqq$$

$$\leqq 4k\gamma_r\sigma^{1-r}(\beta_{1,r} + 1)\, \mathsf{E}\{N^{(3-r)/2} w(f^{(r-1)};\, \sigma^{-1}N^{-1/2})\}/(r-1)!\,.$$

If

$$f^{(r-1)} \in \mathrm{Lip}\,(\alpha, k)\,, \quad 0 < \alpha \leqq 1\,,$$

then

$$|\mathsf{E}f(S_N/\sqrt{N}\sigma) - \int_R f(x)\, \mathrm{d}\Phi(x)| \leqq 4k\gamma_r(\beta_{1,r} + 1)\, \sigma^{1-r-\alpha}\, \mathsf{E}\{N^{(3-r-\alpha)/2}\}\,.$$

Proof. Since $(1/n)\, V_n \to \sigma^2$ a.s., the rest follows from Basu [1980].
In the next theorem the conditions of Theorem 2.1 are weakened, i.e. we restrict

ourselves to the existence of moments of lower orders. Let the absolute pseudo-moment of order r be $v_{k,r} = \int |x|^r \, |d(F_k^*(x) - \Phi_{Y_k}(x))|$ where Φ_{Y_i} is the d.f. of $\sigma_k Y_k$.

Theorem 2.2. *Let $\{X_n\}$ be a sequence of r. vs. satisfying (2.1) and (2.2) for $(2 \leqq i \leqq r-1)$ and $\mathsf{E} v_{k,r-1+\delta} < \infty$ $0 < \delta < 1$ $(1 \leqq k \leqq n)$ for some positive integer $r \geqq 3$. Then for $f \in C_B^{r-1}(R)$ and $f^{(r-1)} \in \mathrm{Lip}(\delta, k)$,*

$$\left|\mathsf{E}[f(S_N/M)] - \int_R f(x)\, \mathrm{d}\Phi(x)\right| \leqq$$

$$\leqq 2\mathsf{E}\{M^{1-r-\delta}(\sum_{k=1}^{N} \mathrm{Max}\,(v_{k,r}^{(r-1)/r}, v_{k,r}))\}/(r-1)! \leqq$$

$$\leqq 2/(r-1)!\, \mathsf{E}\{M^{1-r-\delta} \sum_{k=1}^{N} v_{k,r-1+\delta}\}\,.$$

Proof. The proof is the same as that of Theorem 2.1 except for the following facts (cf. Sakalauskas [1977]):

$$\int |x|^{r-1}(1 + |x|)\, |F_k^* - \Phi_{Y_k}|\, (\mathrm{d}x) \leqq (v_{k,r}^{(r-1)/r} + v_{k,r}) \leqq$$

$$\leqq 2\, \mathrm{Max}\,(v_{k,r}^{(r-1)/r}, v_{k,r})\,,$$

and

$$\frac{1}{(r-1)!} M^{-(r-1)} \int |f^{(r-1)}(\eta) - f^{(r-1)}(y)|\, |(F_k^* - \Phi_{Y_k})\,(\mathrm{d}x)| \leqq$$

$$\leqq \frac{1}{(r-1)!} M^{-(r-1)} \int |x^{r-1}|\, w(f^{(r-1)}, |M^{-1}x|)\, |(F_k^* - \Phi_{Y_k})\,(\mathrm{d}x)| \leqq$$

$$\leqq C/(r-1)!\, M^{-(r-1)} \int |x^{r-1+\delta}|\, |(F_k^* - \Phi_{Y_k})\,(\mathrm{d}x)| \leqq$$

$$\leqq C/(r-1)!\, M^{-(r-1)}\, v_{k,r-1+\delta}\,,$$

where y is such that $|y - \eta| \leqq M^{-1}|x|$.

3. "LOWER CASE o" APPROXIMATION

Theorem 3.1. *Let $\{X_n, n \geqq 1\}$ be a sequence of random variables such that (2.1) and (2.2) hold for some $r \geqq 2$, $(2 \leqq i \leqq r)$, and $\mathsf{E} v_{r,k} < \infty$.*

If $$T_N^r/M^r = O(\mathsf{E}\{T_N^r/M^r\}) \quad \text{a.s. as} \quad \lambda \to \infty\,,$$

(3.1) $$\lim_{\lambda\to\infty} \mathsf{E}(\max_{1 \leqq k \leqq N} \sigma_k^2/M^2) \doteq 0$$

and, for every $\varepsilon > 0$,

(3.2) $$\lim_{\lambda\to\infty} \mathsf{E}\left(T_N^{-r} \sum_{k=1}^{N} \int_{|x| > \varepsilon M} |x|^r \mathrm{d}F_k(x)\right) = 0\,,$$

then, for any $f \in C_B^r(R)$,

$$\left|\mathsf{E}[f(S_N/M)] - \int f(x)\,\mathrm{d}\Phi(x)\right| = o(\mathsf{E}\{T_N^r/M^r\}) \quad \text{as} \quad \lambda \to \infty$$

and also

$$o(E(\sum_{k=1}^{N} v_{k,r}/M^r)) \quad \text{as} \quad \lambda \to \infty\,.$$

Remark. Rychlik and Szynal [1979] calls the condition (3.2) the generalized random Lindeberg condition of order r, and the condition (3.1) the random Feller condition.

Proof. By the same method as used in the preceding theorems and by the consideration of Rychlik and Szynal [1979, Theorem 6], Theorem 3.1 follows.

4. RANDOM VECTORS

Let $R^p = \{x: x = (x^{(1)}, \ldots, x^{(p)})\}$ be the p-dimensional Euclidean space with the scalar product $(x, y) = \sum_{i=1}^{p} x^{(i)} y^{(i)}$ and the norm $\|x\| = (\sum_{i=1}^{p} x^{(i)2})^{1/2}$, and let $X_1, X_2, \ldots$ $\ldots, X_n, \ldots$ be a sequence of $(\mathscr{F}_1 \subseteq \mathscr{F}_2 \subseteq \ldots \subseteq \mathscr{F}_n \subseteq \mathscr{F})\, \mathscr{F}_n$ – measurable p-dimensional random vectors.

Assume

(4.1) $$E(X_k \mid F_{k-1}) = 0 \quad \text{for} \quad k \geqq 2 \quad \text{and} \quad E(X_1) = 0\,.$$

The trace of a matrix Σ is denoted by tr Σ.

Denote by $\Sigma_k = E_{k-1}(X_k X_k')$, the $p \times p$ random matrices, and by $B_k^2 = \operatorname{tr} \Sigma_k$, $k = 1, 2, \ldots, n$, the random variables.

Let $\{Y_k\}$ be a sequence of independent p-variate normal random vectors with p-variate normal distribution $G(x)$ having a mean vector O and a covariance matrix I, and let Y_k'S be independent of $X_k'S$ and the random variables $N = N(\lambda)$.

If $f \in C(R^p)$, $h \in R^p$, $\varepsilon > 0$, then the mod'lus of continuity is defined by $w(f; \varepsilon) = \sup_{\|h\| < \varepsilon} |f(x + h) - f(x)|$. The function $f \in C(R^p)$ satisfies a Lipschitz condition (we write $f \in \operatorname{Lip} \alpha$, if $w(f; \varepsilon) = O(\varepsilon^\alpha)$).

Let $C^r(R^p)$ be the space of all functions bounded and uniformly continuous together with all derivatives up to and including those of rth order. In the following we need Taylor's formula. If $f \in C^r(R^p)$ then we have the so-called Taylor's formula

$$f(x + h) = f(x) + f'(x)(h) + 1/2 f''(x)(h)^2 + \ldots + \frac{1}{r!} f^{(r)}(x + \theta h)(h)^r\,,$$

where $0 < \theta < 1$, $(h)^i = \underbrace{(h, h, \ldots, h)}_{i}$. The norm of the function $f \in C(R^p)$ is defined by $\|f\| = \sup_{x \in R^p} |f(x)|$.

The modulus of continuity satisfies the inequality $w(f; \lambda\varepsilon) \leqq (1 + \lambda)\, w(f; \varepsilon)$ for $\lambda > 0$ (for a proof see Sakalauskas [1977]).

Define $\gamma_r = \int \|x\|^r \,\mathrm{d}G(x)$, $\beta_{r,k} = \mathsf{E}\|x_k\|$, $T_N^r = \sum_{k=1}^{N} \beta_{r,k}$, $V_n^2 = \sum_1^n \operatorname{tr} \Sigma_k$, $M^2 = \sum_1^N \operatorname{tr} \Sigma_k$.

Denote the conditional coordinate pseudomoments of order j by

$$v_k(j) = \sum_{0 \leqq k_1, \ldots, k_j \leqq p} (x_{k_1}^{r_1} \ldots x_{k_j}^{r_j} \,\mathrm{d}[F_k^*(xV_n) - G(xY_n/B_k)]) ,$$

$$r_1 + \ldots + r_j = j$$

and the absolute pseudomoments of order r by

$$v_{k,r} = \int_{R^p} \|x\|^r \,|\mathrm{d}(F_k(x) - G(x))| .$$

Let

$$R_n = \left| \mathsf{E}[f(S_N/M)] - \int_{R^p} f(x) \,\mathrm{d}G(x) \right| .$$

Theorem 4.1. *Let $\{X_k\}$ satisfy (4.1) and suppose that for an integer $r \geqq 3$,*

$$v_k(j) = 0 , \quad 1 \leqq j \leqq r - 1 , \quad k = 1, 2, \ldots, n , \tag{4.2}$$

$$v_{k,r} < \infty , \quad k = 1, 2, \ldots, n , \tag{4.3}$$

and $f \in C^{r-1}(R^p)$. Then

$$R_n \leqq 2\mathsf{E}\{M^{1-r} w(f^{(r-1)}; M^{-1}) \sum_{k=1}^{N} \max(v_{k,r}^{(r-1)/r}, v_{k,r})\}/(r-1)! .$$

If, moreover, we assume that $f^{(r-1)} \in \operatorname{Lip} \alpha$, $0 < \alpha < 1$, then

$$R_n = 0\,(\mathsf{E}\{M^{1-r-\alpha} \sum_{k=1}^{N} \max(v_{k,r}^{(r-1)/r}, v_{k,r})\} .$$

Proof. Define $Z_{n,k} = V_n^{-1}(X_1 + \ldots + X_{k-1}) + (B_{k+1}V_{k+1} + \ldots + B_n Y_k)\, V_n^{-1}$. The rest of the proof is the same as for Theorem 2.1 and hence is omitted.

In the next theorem the conditions of Theorem 4.1 are weakened, i.e. we restrict ourselves to the existence of moments of lower orders.

Theorem 4.2. *Let $\{X_k\}$ satisfy (4.1), and for an integer $r \geqq 3$ suppose that*

$$v_k(j) = 0 , \quad 2 \leqq j \leqq r - 1 , \quad k = 1, 2, \ldots, n , \tag{4.4}$$

$$v_{k,r-1+\alpha} < \infty , \quad 0 < \alpha < 1 , \quad k = 1, 2, \ldots, n ,$$

and

$$f \in C^{r-1}(R^p) , \quad f^{(r-1)} \in \operatorname{Lip} \alpha .$$

Then

$$R_n \leqq 2C/_{(r-1)!}\, \mathsf{E}\{M^{1-r-\alpha} \sum_{k=1}^{N} v_{k,r-1+\alpha}\} .$$

Proof. This theorem can be proved by the same method as used in Theorem 2.2 and hence the proof is omitted.

Let us state a "lower case o" approximation theorem for random vectors without proof.

Theorem 4.3. *Let* $\{X_n, n \geqq 1\}$ *be a sequence of random vectors satisfying* (4.1) *and suppose that for an integer* $r \geqq 2$, $\nu_k(j) = 0$, $1 \leqq j \leqq r$, $k = 1, 2, \ldots, n$. *and* (4.3) *hold.*

If $T_N^r/M^r = 0\left(\mathsf{E}\{T_N^r/M^r\}\right)$ *a.s. for* $\lambda \to \infty$,

$$\operatorname*{Lim}_{\lambda \to \infty} \mathsf{E}\left(\max_{1 \leqq k \leqq N} B_k^2/M^2\right) = 0$$

and, for every $\varepsilon > 0$,

$$(4.5) \qquad \operatorname*{Lim}_{\lambda \to \infty} \mathsf{E}\left(T_N^{-r} \sum_{k=1}^{N} \int_{\|x\| > \varepsilon M} \|x\|^r \, \mathrm{d}F_k(x)\right) = 0\,,$$

then for any $f \in C^r(R^p)$,

$$R_n = o\left(\mathsf{E}\{T_N^r/M^r\}\right) = o\left(\mathsf{E}\left(\sum_{k=1}^{N} \nu_{k,r}\right)/M^r\right) \quad \text{as} \quad \lambda \to \infty\,.$$

Now $B_1^2 = B^2 = \operatorname{Tr} \Sigma_1$ is a.s. a cobstant if $\mathscr{F}_0 = \{\phi, \Omega\}$, and by the ergodic theorem, $(1/n)\, V_n \to B^2$ a.s.

Corollary 4.2. *Let* $\{X_n, n \geqq 1\}$ *be a stationary ergodic sequence of random vectors satisfying* (4.1),

$$(4.6) \quad \nu_k(j) = \sum_{0 \leqq k_1, \ldots, k_j \leqq p} \left(x_{k_1}^{r_1} \ldots x_{k_j}^{r_j} \, \mathrm{d}[F_1^*(xV_n) - G(xV_n/B_k)]\right) = 0, \quad r_1 + \ldots + r_j = j$$

for $2 \leqq j \leqq r - 1$, $1 \leqq k \leqq n$, *and*

$$\nu_{1,r-1+\alpha} = \int_{R^p} \|x\|^{r-1+\alpha} \left|\mathrm{d}(F_1(x) - G(x))\right| < \infty \quad \textit{for} \quad r \geqq 3\,.$$

Then for $f \in C^{r-1}(R^p)$ *and* $f^{(r-1)} \in \operatorname{Lip} \alpha$,

$$\left|\mathsf{E}[f(S_N/\sqrt{N}B)] - \int_{R^p} f(x) \, \mathrm{d}G(x)\right| = 0\left((\mathsf{E}N^{(3-r-\alpha)/2}\right)\,.$$

Corollary 4.3. *Let* $\{X_n, n \geqq 1\}$ *be a stationary ergodic sequence of random vectors satisfying* (4.1), $\nu_k(j) = 0$ *for* $2 \leqq j \leqq r$, $1 \leqq k \leqq n$ *and* $\nu_{1,r} < \infty$ *for* $r \geqq 2$. *If* $N \xrightarrow{P} \infty$ *as* $\lambda \to \infty$ *and* $N^{-(r-2)/2} = 0(\mathsf{E}N^{-(r-2)}/^2)$, a.s. *as* $\lambda \to \infty$, *then for any* $f \in C^r(R^p)$,

$$\left|\mathsf{E}|f(S_N/\sqrt{N}B)\right| - \left.\int_{R^p} f(x) \, \mathrm{d}G(x)\right| = 0(\mathsf{E}\{N^{-(r-2)/2}\}) \quad \text{as} \quad \lambda \to \infty\,.$$

Corollary 4.4. *Let* $\{X_n, n \geqq 1\}$ *be a sequence of random vectors satisfying* (4.1) *and* (4.5) *with* $r = 2$. *Then*

$$\operatorname*{Sup}_x |P[S_N \leqq xM] - G(x)| = o(1) \quad \text{as} \quad \lambda \to \infty\,.$$

Concluding Remark. Generalization to infinite dimensional spaces is straightforward and will be treated elsewhere.

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Souhrn

O ŘÁDU APROXIMACE V CENTRÁLNÍ LIMITNÍ VĚTĚ

A. K. Basu

Autor odvozuje aproximace typu O a o střední hodnoty třídy hladkých funkcí ($f \in C^r(R)$) normovaných náhodných částečných součtů závislých náhodných proměnných pomocí střední hodnoty odpovídajících funkcí Gaussových náhodných proměnných. Tentýž typ aproximace je odvozen také pro závislé náhodné vektory. Tím jsou zobecněny a zlepšeny dřívější výsledky autora (1980) a Rychlika a Szynala (1979).

Резюме

О ПОРЯДКЕ АППРОКСИМАЦИИ В ЦЕНТРАЛЬНОЙ ПРЕДЕЛЬНОЙ ТЕОРЕМЕ

A. K. Basu

Автор выводит аппроксимации типа O и o среднего значения класса гладких функций ($f \in C^r(R)$) нормированных случайных частичных сумм зависимых случайных переменных при помощи среднего значения соответствующих функций Гаусса случайных переменных. Этот же тип аппроксимации выведен также для зависимых случайных векторов. Этим обобщены и улучшены прежние результаты автора (1980) и Рыхлика и Шинала (1979).

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