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CONVERGENCE OF A METHOD FOR SOLVING THE MAGNETOSTATIC FIELD IN NONLINEAR MEDIA

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For a stationary magnetic field at the points of a domain which does not contain the boundary of various media we have a set of Maxwell equations in differential form:

$$\text{rot}\,\boldsymbol{H} = \boldsymbol{J} \tag{1}$$

$$\text{div}\,\boldsymbol{B} = 0 \tag{2}$$

where $\boldsymbol{B}$ is the magnetic plane-density vector of magnitude $\boldsymbol{B}$, $\boldsymbol{H}$ is the magnetic field-intensity vector of magnitude $\boldsymbol{H}$, $\boldsymbol{J}$ is the current-density vector of magnitude $\boldsymbol{J}$.

For magnetic isotropic medium we have

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{3}$$

where μ is a scalar quantity called permeability of the medium. μ is constant with respect to H in magnetic linear media; for the vacuum and approximately for the air the permeability is $\mu_0 = 4\pi \,.\, 10^{-7} Hm^{-1}$. In magnetic nonlinear media (ferromagnetics) μ is a function of H:

$$\mu = \mu(H) \tag{4}$$

The introduced set of Maxwell equations in differential form is completed by the boundary conditions at the points of a boundary:

$$H_{1\tau} = H_{2\tau} \tag{5}$$

$$\mu_1 H_{1\nu} = \mu_2 H_{2\nu} \tag{6}$$

where τ denotes the projection of the vector $\boldsymbol{H}$ into a tangent and ν denotes its projection into a normal of the boundary at a point considered.

To this purpose we shall restrict ourselves to solving a stationary magnetic field in a two-dimensional simply connected domain Ω of a ferromagnetic of the permeability $\mu(H)$ assuming the current-density J to be zero at all points of the domain Ω. Let (x, y) denote the cartesian coordinates in Ω. Then according to (1) we can suppose

$$\mathbf{H} = -\operatorname{grad} \varphi \tag{7}$$

where the scalar function φ is called the scalar magnetic potential. Applying (3) and (5) to the equation (2) we obtain

$$\operatorname{div} \mu \operatorname{grad} \varphi = 0 , \tag{8a}$$

i.e. in given cartesian coordinates (x, y),

$$\frac{\partial}{\partial x}\left(\mu \frac{\partial \varphi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial \varphi}{\partial y}\right) = 0 \tag{8}$$

or

$$\Delta \varphi = \frac{1}{\mu} \mathbf{H} \operatorname{grad} \mu , \tag{8b}$$

where the symbol Δ denotes the two-dimensional Laplace operator and the symbol grad denotes the two-dimensional gradient.

For solving this nonlinear partial differential equation with the mixed boundary conditions on the boundary Γ of the domain Ω:

$$\begin{aligned} \varphi &= f(x, y) \quad \text{on} \quad \Gamma_1 \\ \mu \frac{\partial \varphi}{\partial \nu} &= g(x, y) \quad \text{on} \quad \Gamma_2 \end{aligned} \tag{9}$$

where $f(x, y)$, $g(x, y)$ are given functions, $\Gamma_1 \cup \Gamma_2 = \Gamma$ (Γ_1 having non-zero length), it is possible to use a linearization based on the successive approximations as follows:

$$\begin{aligned} \Delta \varphi_1 &= 0 , \\ \Delta \varphi_k &= \frac{1}{\mu_{k-1}} \mathbf{H}_k \operatorname{grad} \mu_{k-1} , \quad k = 2, 3, \ldots \end{aligned} \tag{10}$$

for mixed boundary conditions on Γ:

$$\begin{aligned} \varphi_k &= f(x, y) \qquad \text{on } \Gamma_1 \\ \mu_{k-1} \frac{\partial \varphi_k}{\partial \nu} &= g(x, y) \qquad \text{on } \Gamma_2 \end{aligned} \tag{11}$$

Application of this method needs, however, to make clear the question concerning the condition of a solution of linear boundary problem (10), (11) and convergence to a solution of the boundary problem (8), (9). As far as we know, this question was not fully theoretically solved, because of its difficulty. The answer depends obviously on the properties of function $\mu(H)$ which are given by physical conditions under which the above mentioned method of successive approximations comes true. Practically the dependence $B(H)$ given by a hysteresis loop is approximated by average course $B(H)$, (Fig. 1), so that

(12) $\mu(H)$ is non-increasing,

(13) $\mu(H)$ is bounded:

$$0 < \mu_0 < \mu < K$$

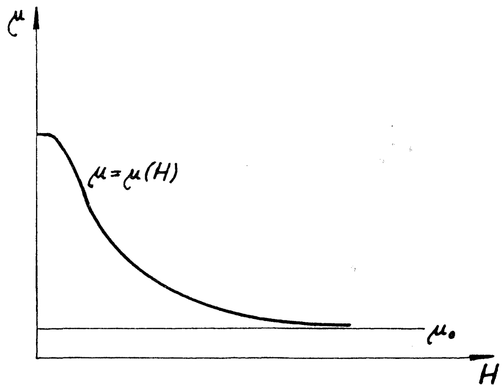

Fig. 1.

where μ_0 is permeability of vacuum, K is a constant,

$$\mu_d = \frac{dB}{dH} = \mu + H \frac{d\mu}{dH} > 0\,, \quad \lim_{H \to \infty} \mu_d = \mu_0\,. \tag{14}$$

The purpose of this paper is to prove that under these physical conditions the convergence of the introduced successive approximations for the solution of boundary problem (8), (9) is always ensured.

For simplicity let us denote $m(\alpha) = \mu(\sqrt{\alpha})$, $\alpha \in \langle 0, \infty)$ and assume that $m(\alpha)$ is from $C^1 \langle 0, \infty)$ and, in accordance with (12)–(14) fulfils the following conditions

(15) $m(\alpha)$ is non-increasing

$$0 < \mu_0 < m(\alpha) < K < \infty\,, \quad 0 \leqq \alpha < \infty \tag{16}$$

(17) there exists a constant $c > 0$ such that $2\alpha\, m'(\alpha) + m(\alpha) > c$.

(18) Let us denote $M(\alpha) = \int_0^\alpha m(\beta)\, d\beta$.

(19) Let Ω be a bounded domain in E_2 with Lipschitzian boundary $\partial\Omega$. Let $\partial\Omega = \Gamma_1 \cup \Gamma_2 \cup N$, $\Gamma_1 \cap \Gamma_2 = 0$ where Γ_1, Γ_2 are open sets in $\partial\Omega$ and the one-dimensional measure of Γ_1 is positive and the measure of N is zero.

A set of real functions having continuous partial derivatives of any order in $\bar{\Omega}$ will be denoted by $\varepsilon(\Omega)$.

Let supp ψ be the closure of the set $\{x; x \in \bar{\Omega}, \psi(x) \neq 0\}$, where $\psi(x)$ is defined on $\bar{\Omega}$.

$$\|\psi\|_{W_2^{(1)}(\Omega)} = \sqrt{\int_\Omega \left(|\psi|^2 + \left|\frac{\partial \psi}{\partial x}\right|^2 + \left|\frac{\partial \psi}{\partial y}\right|^2 \right) \mathrm{d}\Omega} \,.$$

Let us write $\mathfrak{V} = \{\psi \in \mathscr{E}(\Omega); \varrho(\operatorname{supp} \psi, \Gamma_1) > 0\}$ where $\varrho(x, y)$ is a distance of the points x, y.

Let V be the closure of $\mathfrak{V}$ in the norm $W_2^{(1)}(\Omega)$.

Let $f(s)$, $g(s)$ be the functions from $L_2(\partial\Omega)$ and suppose that there exists $\varphi^* \in W_2^{(1)}(\Omega)$ such that $\varphi^*|_{\partial\Omega} = f$. (Whenever f satisfies Lipschitz condition on $\partial\Omega$, φ^* exists.)

We shall look for the weak solution of the boundary value problem (8), (9) i.e. $u \in W_2^{(1)}(\Omega)$ is a solution of the problem I if (20) $u - \varphi^* \in V$ and for each $\psi \in V$,

$$\int_0 m(u_x^2 + u_y^2)\,(u_x\psi_x + u_y\psi_y)\,\mathrm{d}\Omega - \int_{\Gamma_2} \psi g \,\mathrm{d}s = 0$$

where $u_x = \partial u/\partial x$ e.t.c.

For simplicity we shall write $m(u) = m(u_x^2 + u_y^2)$,

$$M(u) = M(u_x^2 + u_y^2)\,, \quad m'(u) = m'(u_x^2 + u_y^2)\,, \quad u \in W_2^{(1)}(\Omega)\,.$$

Furthermore, because of the properties of $m(\alpha)$, the following relations are also satisfied:

(21) $$|m(\varphi)\,\varphi_x| \leqq K|\varphi_x|\,, \quad |m(\varphi)\,\varphi_y| \leqq K|\varphi_y|$$

(22) $$\frac{\partial(m(\varphi)\,\varphi_x)}{\partial \varphi_y} = \frac{\partial(m(\varphi)\,\varphi_y)}{\partial \varphi_x}\,.$$

A functional $\Phi(u)$ defined on E (E being a Banach space) has a linear Gateaux' differential $D\,\Phi(u, v)$ at a point u, if for each $v \in E$ there exists

$$\lim_{t \to 0} \frac{\Phi(u + tv) - \Phi(u)}{t}$$

and this limit is linear with respect to v.

According to the work of M. M. Vajnberg [3] and J. Nečas [2] it follows – using the fact that $m(\alpha) \in C^1\langle 0, \infty)$ and that (21), (22) hold – that there exists a corresponding functional to the problem I and has the form

$$\int_0^1 \mathrm{d}t \int_\Omega \{m(\varphi^* + t(u - \varphi^*)) [(\varphi_x^* + t(u_x - \varphi_x^*)) (u_x - \varphi_x^*) + (\varphi_y^* + t(u_y - \varphi_y^*)) (u_y - \varphi_y^*)]\} \, \mathrm{d}\Omega .$$

Because of (18) this functional can be rewritten in the form

$$\frac{1}{2}\int_\Omega M(u) \, \mathrm{d}\Omega - \frac{1}{2}\int_\Omega M(\varphi^*) \, \mathrm{d}\Omega - \int_{\Gamma_2} gu \, \mathrm{d}s + \int_{\Gamma_2} g\varphi^* \, \mathrm{d}s$$

and adding a suitable constant we obtain

$$\Phi(u) = \frac{1}{2}\int_\Omega M(u) \, \mathrm{d}\Omega - \int_{\Gamma_2} gu \, \mathrm{d}s \,, \quad u - \varphi^* \in V . \tag{23}$$

From the construction of the functional (23) it is clear that

$$D \, \Phi(u, \psi) = 0 \,, \tag{24}$$

i.e. just the same as (20).

Let us compute the second Gateaux differential $D^2 \, \Phi(u, h, h)$ for $u - \varphi^* \in V$, $h \in V$. Using (18), (15), (17) and Schwarz's inequality we obtain:

$$D^2 \, \Phi(u, h, h) = \int_\Omega [m(u) (h_x^2 + h_y^2) + 2m'(u) (u_x h_x + u_y h_y)^2] \, \mathrm{d}\Omega \geqq$$
$$\geqq \int_\Omega [m(u) + 2m'(u) (u_x^2 + u_y^2)] (h_x^2 + h_y^2) \, \mathrm{d}\Omega \geqq c \int_\Omega (h_x^2 + h_y^2) \, \mathrm{d}\Omega \,,$$

$$\left[\int_\Omega (h_x^2 + h_y^2) \, \mathrm{d}\Omega\right]^{\frac{1}{2}} \tag{25}$$

forms an equivalent norm in V with respect to (19) (see e.g. J. NEČAS [1]).

Therefore there exists a constant $c_1 > 0$ such that

$$D^2 \, \Phi(u, h, h) \geqq c_1 \|h\|^2_{W_2^{(1)}} . \tag{26}$$

In accordance with the results of J. NEČAS [2] and (21), (22), (26) being fulfilled, there exists only one solution u_0 of problem I, for which

$$\min_{u - \varphi^* \in V} \Phi(u) = \Phi(u_0) . \tag{27}$$

Now let us consider the linearization of the problem I. Let $v \in W_2^{(1)}(\Omega)$, $v - \varphi^* \in V$ be fixed. Then $u \in W_2^{(1)}(\Omega)$ is a solution of the problem II if $u - \varphi^* \in V$ and for each $\psi \in V$

$$\int_\Omega m(v) (u_x \psi_x + u_y \psi_y) \, \mathrm{d}\Omega - \int_{\Gamma_2} \psi g \, \mathrm{d}s = 0 \tag{28}$$

holds.

The corresponding functional to the problem II – obtained similarly as in the problem I – is the following (the additive constant being omitted):

$$(29) \qquad P_v(u) = \frac{1}{2}\int_\Omega [M(v) - m(v)(v_x^2 + v_y^2) + m(v)(u_x^2 + u_y^2)]\, d\Omega - \\ - \int_{\Gamma_2} gu\, ds\,, \quad u \in W_2^{(1)}(\Omega)\,, \quad u - \varphi^* \in V\,.$$

By the same argument as in I one can see that the problem II has only one solution for which the functional P_v attains its minimum.

Let us take an arbitrary $u_1 \in W_2^{(1)}(\Omega)$, $u_1 - \varphi^* \in V$ and let us define a sequence u_k, $k = 2, 3, \ldots$ in the following way

(30) u_k is a solution of problem II for $v = u_{k-1}$, $k = 2, 3, \ldots$

By the properties of P_v it follows

$$(31) \qquad \min_{u - \varphi^* \in V} P_{u_{k-1}}(u) = P_{u_{k-1}}(u_k)\,.$$

Theorem 1. *The sequence of u_k, $k = 1, 2, \ldots$ from (30) which solves the linear problem II converges in the space $W_2^{(1)}(\Omega)$ to a function u_0 which solves the non-linear problem I.*

Proof. From (29) and (23) we easily find that

$$(32) \qquad P_u(u) = \Phi(u)$$

holds for each $u - \varphi^* \in V$.

We shall prove the following inequality

$$(33) \qquad P_{u_n}(u_{n+1}) \geqq \Phi(u_{n+1})\,, \quad n = 1, 2, \ldots$$

From (15), (18) one can see that M is concave and therefore for a, $b \geqq 0$,

$$(34) \qquad M(a) - M(b) \geqq m(a)(a - b)\,.$$

Table 1

296·8					
287·5	290·6				
270·2	273·3	281·3			
245·0	247·8	254·2	267·8		
212·8	215·2	220·3	229·4	247·9	
175·1	177·0	180·8	187·2	197·7	213·9
133·8	135·0	137·6	141·6	147·0	152·5
90·1	90·8	92·4	94·7	97·3	99·5
45·2	45·6	46·3	47·4	48·5	49·3

Then we substitute $a = u_{n,x}^2(z) + u_{n,y}^2(z)$, $b = u_{n+1,x}^2(z) + u_{n+1,y}^2(z)$ for each $z \in \Omega$. Considering the expression $P_{u_n}(u_{n+1}) - \Phi(u_{n+1})$ with respect to (23), (29) and using (34) we find that (33) holds.

If we put together relations (31), (32) and (33) we obtain

$$(35) \qquad \Phi(u_n) = P_{u_n}(u_n) \geqq P_{u_n}(u_{n+1}) \geqq \Phi(u_{n+1}), \quad n = 1, 2, \ldots$$

Now we substitute $v = u_n$, $u = u_{n+1}$ and $\psi = u_{n+1} - u_n \in V$ into relation (28)

$$(36) \qquad \int_\Omega m(u_n)\,(u_{n+1,x}^2 + u_{n+1,y}^2)\,d\Omega - \int_{\Gamma_2} g u_{n+1}\,ds =$$
$$= \int_\Omega m(u_n)\,(u_{n+1,x}u_{n,x} + u_{n+1,y}u_{n,y})\,d\Omega - \int_{\Gamma_2} g u_n\,ds\,.$$

From (36), (23) and (29) we obtain

$$(37) \qquad \int_\Omega m(u_n)\,[(u_{n+1,x} - u_{n,x})^2 + (u_{n+1,y} - u_{n,y})^2]\,d\Omega =$$
$$= 2[\Phi(u_n) - P_{u_n}(u_{n+1})] \leqq 2[\Phi(u_n) - \Phi(u_{n+1}]\,.$$

In accordance with (16), (25) and (37) there exists a constant $c_2 > 0$ such that $c_2\|u_{n+1} - u_n\|^2_{W_2^{(1)}} \leqq \Phi(u_n) - \Phi(u_{n+1})$. Using (35) and (27) we have

$$(38) \qquad \|u_{n+1} - u_n\|_{W_2^{(1)}} \to 0\,, \quad n \to \infty\,.$$

From the definition of u_0 and (24), $D\,\Phi(u_0, u_n - u_0) = 0$. Using Lagrange's theorem and (26) there exists $\vartheta \in (0, 1)$ such that

$$(39) \qquad D\,\Phi(u_n, u_n - u_0) = D\,\Phi(u_n, u_n - u_0) - D\,\Phi(u_0, u_n - u_0) =$$
$$= D^2\,\Phi(u_0 + \vartheta(u_n - u_0), u_n - u_0, u_n - u_0) \geqq c_1\|u_n - u_0\|^2_{W_2^{(1)}}\,.$$

For $h \in V$ we have according to (24), (28) and (16)

$$D\,\Phi(u_n, h) = \int_\Omega m(u_n)\,(u_{n,x}h_x + u_{n,y}h_y)\,d\Omega - \int_{\Gamma_2} gh\,ds -$$
$$- \int_\Omega m(u_n)\,(u_{n+1,x}h_x + u_{n+1,y}h_y)\,d\Omega + \int_{\Gamma_2} gh\,ds =$$
$$= \int_\Omega m(u_n)\,[(u_{n,x} - u_{n+1,x})\,h_x + (u_{n,y} - u_{n+1,y})\,h_y]\,d\Omega \leqq$$
$$\leqq K\|u_n - u_{n+1}\|_{W_2^{(1)}}\,\|h\|_{W_2^{(1)}}\,.$$

Let us substitute $h = u_n - u_0$. From (39) we obtain

$$c_1\|u_n - u_0\|^2_{W_2^{(1)}} \leqq K\|u_n - u_{n+1}\|_{W_2^{(1)}}\,\|u_n - u_0\|_{W_2^{(1)}}\,.$$

Thus we have with respect to (24)

$$\|u_n - u_0\|_{W_2^{(1)}} \to 0 \quad \text{for} \quad n \to \infty \text{ q.e.d.}$$

From the results of De Giorgi [4] and according to (16) it is possible to change function u_n and u_0 on a set of measure zero and then u_0, u_n will be uniformly Hölderian with coefficients τ, α on each inner subset C of domain Ω, i.e.

$$|u_n(x_1) - u_n(x_2)| \leqq \tau |x_1 - x_2|^\alpha, \quad (x_1, x_2 \in C, 0 < \alpha < 1, n = 1, 2, \ldots)$$
$$|u_0(x_1) - u_0(x_2)| \leqq \tau |x_1 - x_2|^\alpha$$

where α depends on $C \subset \Omega$ and on the number K/μ_0. τ depends on $\sup_C |u_n|$ and $\sup_C |u_0|$.

Following J. Nečas [1] the numbers $\sup_C |u_n|$, $\sup_C |u_0|$ are bounded from above by some constant depending only on C.

Theorem 2. *$u_n \to u_0$ on every closed domain C, which is an inner subset of Ω.*

Proof. (By contradiction.) Let $a \in C$ exist such that $u_n(a)$ does not converge to $u_0(a)$. Then there exists $\varepsilon > 0$ and subsequence u_{n_k} such that $u_{n_k}(a) \geqq u_0(a) + \varepsilon$ (or $u_{n_k}(a) \leqq u_0(a) - \varepsilon$). Functions u_{n_k} and u_0 are uniform by Hölderian and therefore there exists a number ϱ_0 such that

$$x \in C, \quad \varrho(x, a) \leqq \varrho_0 \Rightarrow |u_{n_k}(x) - u_{n_k}(a)| \leqq \frac{\varepsilon}{4}, \quad |u_0(x) - u_0(a)| \leqq \frac{\varepsilon}{4}.$$

Then $|u_{n_k}(x) - u_0(x)| \geqq \varepsilon/2$ on the set $\sigma = \{x \in C; \varrho(x, a) \leqq \varrho_0\}$.

Therefore

$$\int_\Omega |u_{n_k}(x) - u_0(x)|^2 \, d\Omega \geqq \int_\sigma |u_{n_k}(x) - u_0(x)|^2 \, d\Omega \geqq \left(\frac{\varepsilon}{2}\right)^2 \mu(\sigma) > 0$$

where $\mu(\sigma)$ means the measure of the set σ.

However, this means that u_{n_k} does not converge to u_0 in $L_2(\Omega)$ not even in $W_2^{(1)}(\Omega)$ and this contradicts Theorem 1. q.e.d.

Remark. The method that has been used here can be applied to some other nonlinear problems. In particular, it can be used for solving a problem similar to that one (8), (9) with more variables and with a nonzero right-hand side depending only on the space variables.

The considered process of solving the boundary value problem (8), (9) expressed by the relations (10), (11) can be numerically realized by using electrical analog e.g. an electrolytic tank [5; 6] or a computer. The problem will be illustrated by numerical calculation on the computer ODRA.*)

*) The autors thank for the cooperation in the computation to ing. V. Holub from VŠSE Plzeň.

The boundary conditions on the boundary of a domain (see Fig. 2) are: $\varphi = 0$ on AB, $\varphi = 600$ A on DE (thus $\varphi = 300$ A on CF) and $\partial\varphi/\partial V$ on the rest of Γ. The dependence of μ on H: $\mu = \mu(H)$ for the material in question (tins 2, 6, W/kg) was approximated to the form illustrated in Fig. 1. From the practical point of view already the 3-rd approximation φ_3, μ_3 gave the satisfactory result in correspondence with the results, obtained by measurement [6]. With the necessary accuracy $0{\cdot}1A$ in the values φ_k the coincidence between the 8-th and 9-th approximation has been reached. The resulting values $\varphi_9(A)$ at the points of field which are designated by $1-39$ are given in the tab. 1 and the corresponding values $\mu_8 \cdot 10^{-5}$ (Hm^{-1}) in the Tab. 2.

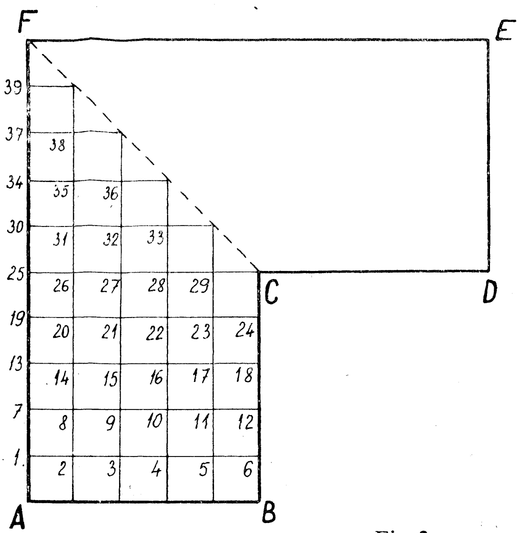

Fig. 2.

Table 2

150·5					
86·4	77·4				
61·2	58·5	48·4			
49·1	47·8	43·9	33·0		
42·6	41·8	39·9	35·5	23·6	
39·1	38·5	37·3	35·0	31·2	27·4
37·3	36·8	36·0	34·7	33·0	31·3
36·4	36·1	35·5	34·6	33·7	32·9
36·1	35·8	35·3	34·6	33·9	33·4

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Souhrn

KONVERGENCE JEDNÉ METODY ŘEŠENÍ MAGNETOSTATICKÉHO POLE V NELINEÁRNÍM PROSTŘEDÍ

Jozef Kačúr, Jindřich Nečas, Josef Polák, Jiří Souček

K řešení okrajového problému pro potenciál rovinného magnetostatického pole ve feromagnetiku (8), (9) lze s výhodou užít linearizace metodou postupných aproximací (10), (11). V článku se dokazuje konvergence této metody, jsou-li splněny podmínky (12), (13), (14).

Резюме

СХОДИМОСТЬ ОДНОГО МЕТОДА РЕШЕНИЯ СТАТИЧЕСКОГО МАГНИТНОГО ПОЛЯ В НЕЛИНЕЙНОЙ СРЕДЕ

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Для решения краевой задачи для потенциала плоскопараллельного статического магнитного поля в ферромагнетике (8), (9) можно с успехом использовать линеаризацию методом последовательных приближений (10), (11). В статье приведено доказательство сходимости этого метода при выполнении условий (12), (13), (14).

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