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TWO - STRIP RHEOLOGICAL MODEL FOR SIMPLE
ORTHOTROPIC VISCOELASTIC BODIES

[PRELIMINARY COMMUNICATION]

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The complexity of the two-dimensional rheological behavior of anisotropic viscoelastic bodies needs, in some cases, a special representation. Therefore, the author has introduced the two-dimensional rheological models, of which the two-strip model on Fig. 1 for orthotropic viscoelastic bodies is the most simple. It consists of the Hookean elastic zone *H* and the Newtonian viscous zone *N*. In the direction of the *x*-axis, it behaves like a Kelvin solid and in the direction of the *y*-axis like a Maxwell liquid.

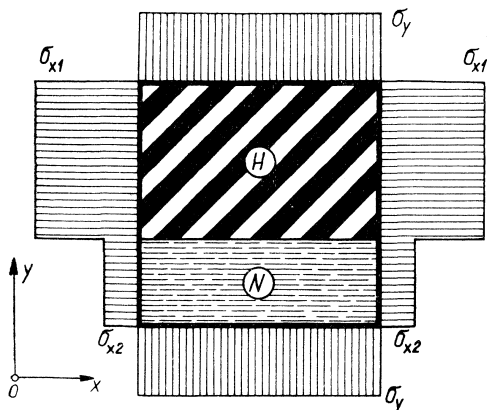


Fig. 1. Two-Strip Rheological Model for a Viscoelastic Body.

The stress σ_x is decomposed into the component σ_{x1} acting in the elastic zone and into σ_{x2} acting in the viscous zone.

The normal strain rates at the plane-stress state are given by

$$(1) \quad \frac{d\epsilon_x}{dt} = \frac{1}{E_x} \left(\frac{d\sigma_{x1}}{dt} - \mu_x \frac{d\sigma_y}{dt} \right) = \frac{1}{\lambda_x} (\sigma_{x2} - \nu_x \sigma_y),$$

$$(2) \quad \frac{d\epsilon_y}{dt} = \frac{1}{E_y} \left(\frac{d\sigma_y}{dt} - \mu_y \frac{d\sigma_{x1}}{dt} \right) + \frac{1}{\lambda_y} (\sigma_y - \nu_y \sigma_{x2}),$$

where λ_x, λ_y are the coefficients of normal viscosity,

μ_x, μ_y the elastic Poisson ratios,

ν_x, ν_y the viscous Poisson ratios.

Introducing $\sigma_{x2} = \sigma_x - \sigma_{x1}$ into (1),

$$(3) \quad \frac{d\sigma_{x1}}{dt} + \frac{E_x}{\lambda_x} \sigma_{x1} = \frac{E_x}{\lambda_x} (\sigma_x - \nu_x \sigma_y) + \mu_x \frac{d\sigma_y}{dt},$$

and hence

$$(4) \quad \sigma_{x1} = e^{-(E_x t / \lambda_x)} \left\{ \int_0^t \left[\frac{E_x}{\lambda_x} (\sigma_x - \nu_x \sigma_y) + \mu_x \frac{d\sigma_y}{d\tau} \right] e^{E_x \tau / \lambda_x} d\tau + \sigma_{x10} \right\}.$$

Substituting (4) into (1) and performing the integration, one obtains

$$(5) \quad \varepsilon_x = e^{-(E_x t / \lambda_x)} \left[\int_0^t \left(\frac{\sigma_x - \nu_x \sigma_y}{\lambda_x} + \frac{\mu_x}{E_x} \frac{d\sigma_y}{d\tau} \right) e^{E_x \tau / \lambda_x} d\tau + \frac{\sigma_{x10}}{E_x} \right] - \frac{\mu_x}{E_x} \sigma_y,$$

i.e.

$$(6) \quad \varepsilon_x = e^{-(E_x t / \lambda_x)} \left\{ \int_0^t \frac{1}{\lambda_x} [\sigma_x - (\mu_x + \nu_x) \sigma_y] e^{E_x \tau / \lambda_x} d\tau + \frac{\sigma_{x10}}{E_x} \right\},$$

which also represents the solution of the following first-order linear differential equation

$$(7) \quad \lambda_x \frac{d\varepsilon_x}{dt} + E_x \varepsilon_x = \sigma_x - (\mu_x + \nu_x) \sigma_y,$$

corresponding to the rheological equation of a Kelvin solid.

Introducing

$$(8) \quad \sigma_{x1} = E_x \varepsilon_x + \mu_x \sigma_y,$$

obtained from (1), into (2), one obtains, after integration,

$$(9) \quad \varepsilon_y = \frac{1 - \mu_x \mu_y}{E_y} \sigma_y - \frac{\mu_y E_x}{E_y} \varepsilon_x + \frac{1}{\lambda_x} \int_0^t [(1 + \mu_x \nu_y) \sigma_y - \nu_y \sigma_x + \nu_x E_x \varepsilon_x] d\tau.$$

After substituting from (6), this equation becomes

$$(10) \quad \varepsilon_y = \frac{1 - \mu_x \mu_y}{E_y} \sigma_x - \frac{\mu_y E_x}{E_y} e^{-(E_x t / \lambda_x)} \left\{ \int_0^t \frac{1}{\lambda_x} [\sigma_x - (\mu_x + \nu_x) \sigma_y] e^{E_x \tau / \lambda_x} d\tau + \frac{\sigma_{x10}}{E_x} \right\} + \frac{1}{\lambda_x} \int_0^t \left((1 + \mu_x \nu_y) \sigma_y - \nu_y \sigma_x + \nu_x E_x e^{-(E_x \tau / \lambda_x)} \left\{ \int_0^\tau \frac{1}{\lambda_x} [\sigma_x - (\mu_x + \nu_x) \sigma_y] \cdot e^{E_x \tau' / \lambda_x} d\tau' + \frac{\sigma_{x10}}{E_x} \right\} \right) d\tau.$$

From (7)

$$(11) \quad \sigma_x = E_x \varepsilon_x + \lambda_x \frac{d\varepsilon_x}{dt} + (\mu_x + \nu_x) \sigma_y;$$

introducing this into (9) yields after differentiation,

$$(12) \quad \frac{d\sigma_y}{dt} + \frac{(1 - \nu_x \nu_y) E_y}{(1 - \mu_x \mu_y) \lambda_y} \sigma_y = \frac{E_y}{1 - \mu_x \mu_y} \left[\frac{d\varepsilon_y}{dt} + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y} \right) \frac{d\varepsilon_x}{dt} \right],$$

corresponding to the rheological equation of a Maxwell liquid; hence

$$(13) \quad \sigma_y = e^{-C_y t} \left\{ \frac{E_y}{1 - \mu_x \mu_y} \int_0^t \left[\frac{d\varepsilon_y}{d\tau} + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y} \right) \frac{d\varepsilon_x}{d\tau} \right] e^{C_y \tau} d\tau + \sigma_{y0} \right\},$$

i. e.

$$(14) \quad \sigma_y = \frac{E_y}{1 - \mu_x \mu_y} \left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y} \right) \varepsilon_x \right] - \left(\frac{E_y}{1 - \mu_x \mu_y} \right)^2 \frac{1 - \nu_x \nu_y}{\lambda_y} e^{-C_y t} \int_0^t \left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y} \right) \varepsilon_x \right] e^{C_y \tau} d\tau + \sigma_{y0} e^{-C_y t},$$

where $C_y = (1 - \nu_x \nu_y) E_y / (1 - \mu_x \mu_y) \lambda_y$ is the reciprocal relaxation time.

Substituting (14) into (11) yields

$$(15) \quad \sigma_x = E_x \varepsilon_x + \frac{(\mu_x + \nu_x) E_y}{1 - \mu_x \mu_y} \left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y} \right) \varepsilon_x \right] + \lambda_x \frac{d\varepsilon_x}{dt} - (\mu_x + \nu_x) \cdot \left(\frac{E_y}{1 - \mu_x \mu_y} \right)^2 \frac{1 - \nu_x \nu_y}{\lambda_y} e^{-C_y t} \int_0^t \left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y} \right) \varepsilon_x \right] e^{C_y \tau} d\tau + \sigma_{y0} e^{-C_y t}.$$

The assumption of full compactness of the two-strip model yields, for the symmetrical shear stress and strain, the following expression

$$(16) \quad \sigma_{xy} = \sigma_{yx} = G \varepsilon_{xy} + \eta \frac{d\varepsilon_{xy}}{dt}.$$

If however, the vertical boundaries of the model on Fig. 1 are free to deflect, the body behaves at the vertical shear like a Kelvin solid and the shear stress σ_{xy} is given by (16); for the horizontal shear, the equation of the Maxwell liquid is valid

$$(17) \quad \frac{d\sigma_{yx}}{dt} + \frac{G}{\eta} \sigma_{yx} = G \frac{d\varepsilon_{yx}}{dt}$$

from which

$$(18) \quad \sigma_{yx} = e^{-(Gt/\eta)} \left(\int_0^t G \frac{d\varepsilon_{yx}}{d\tau} e^{G\tau/\eta} d\tau + \sigma_{yx0} \right).$$

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