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IDENTITIES AND DELETING MAPS ON QUASIGROUPS

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V. D. Belousov in [2] and M. A. Taylor in [6] have proved a theorem that is a generalization of a theorem of Belousov (see Theorem 2.1.1 in [3]). In this paper we give a generalization of these results and its applications.

1. PRELIMINARIES

An algebra $(Q, f_1, \ldots, f_n) = (Q, F)$ is called an *algebra of quasigroups* if (Q, f_i) is a quasigroup for all $i \in \{1, 2, \ldots, n\}$. An algebra (Q, f_1, f_2, f_3) is called a *primitive quasigroup* if there exists a permutation (ijk) of the set $\{1, 2, 3\}$ such that (Q, f_j) and (Q, f_k) are respectively the left and the right division groupoids of a quasigroup (Q, f_i); if f_i is denoted by $\cdot$, then we put $f_j = /$, $f_k = \backslash$, thus $(Q, \cdot, /, \backslash)$ means a primitive quasigroup. An algebra (Q, F) is called an *algebra of primitive quasigroups* if for each $f \in F$ there exist $g, h \in F$ such that (Q, f, g, h) is a primitive quasigroup. For every quasigroup (Q, f) there exists a primitive quasigroup (Q, f, g, h) and for every algebra of quasigroups (Q, F) there exists an algebra of primitive quasigroups (Q, G) with $F \subset G$.

Let (Q, A) be a quasigroup; we define $A[x, y] = z$ iff $A^{-1}[x, z] = y$ iff ${}^{-1}A[z, y] = x$ iff ${}^{-1}(A^{-1})[y, z] = x$ iff $({}^{-1}A)^{-1}[z, x] = y$ iff $A^*[y, x] = z$. The set $\{A, {}^{-1}A, A^{-1}, {}^{-1}(A^{-1}), ({}^{-1}A)^{-1}, A^*\} = \Sigma A$ is called the *system of division operations of* A. An algebra (Q, F) is said to be an *algebra of parastrophic quasigroups* if $\Sigma f \subset F$ for each $f \in F$. For every algebra of quasigroups (Q, F) there exists exactly one algebra of parastrophic quasigroups $(Q, \Sigma F)$, where $\Sigma F = \bigcup\{\Sigma f; f \in F\}$.

Throughout the paper, for a quasigroup $(Q, \cdot)$ we put $L_a x = a . x$, $R_a x = x . a$, $T_a x = x \backslash a$, $L_a^{-1} x = a \backslash x$, $R_a^{-1} x = x / a$, $T_a^{-1} x = a / x$,

$$T_0 = \{L, R, T, L^{-1}, R^{-1}, T^{-1}\} .$$

If a quasigroup operation is denoted by another symbol, say $\circ$, then we put $a \circ x =$

$= L_a^\circ x,\ x \circ a = R_a^\circ x, \ldots, T_\circ^\circ = \{L^\circ, R^\circ, \ldots\}$. If (Q, F) is an algebra of quasigroups we denote $T_\circ^F = \bigcup\{T_\circ^\circ;\ \circ \in F\}$.

A word on algebra $(Q, f_1, f_2, \ldots, f_n)$ is a formal expression consisting of variables, brackets and operations $f_1, \ldots, f_n$. The length $l(w)$ of a word w is the number of occurences of variables in w. In the following, the set of all variables that occur in a word w will be denoted by $V(w)$.

Let $x, y, \ldots, z, x_1, \ldots, x_n, \ldots$ be variables and let $a_1, \ldots, a_n, \ldots$ be elements of a set Q. A *retraction map* with invariant variables $x, y, \ldots, z$ (or a retraction map, if there is no danger of confusion) is a map $w \mapsto w_1$, where $V(w) \subset \{x, y, \ldots, z, x_1, \ldots, x_n, \ldots\}$ and w_1 is a formal expression obtained from a word w on an algebra (Q, F) by replacing each variable x_i in w by a_i, for all $x_i \in V(w)$; if a set of invariant variables of a retraction map is empty then the image of a word w is an element of Q.

An *identity* on an algebra (Q, F) is a pair (w, w') of words on (Q, F) that is written $w \doteq w'$. We say that an identity $w \doteq w'$ is valid on (Q, F) (or (Q, F) satisfies $w \doteq w'$) and write $w = w'$ if for every retraction map ϱ with no invariant variables $\varrho w = \varrho w'$, i.e. $\varrho w, \varrho w'$ are equal elements in Q. Words w, w' on (Q, F) are said to be *equivalent* if (Q, F) saitsfies the identiy $w \doteq w'$. A word w_1 is said to be a *subword of an identity* $w \doteq w'$ if w_1 is a subword of w or w'. Identities $w \doteq w', w_1 \doteq w_1'$ are called *equivalent* if the validity of one of them implies the validity of the other. Let ϱ be a retraction map and let w, w' be words on an algebra (Q, F). We say that $\varrho w, \varrho w'$ are equivalent and write $\varrho w \doteq \varrho w'$ or $\varrho w = \varrho w'$ if for each retraction map σ with no invariant variables $\sigma w = \sigma w'$.

An identity $w \doteq w'$ on an algebra (Q, F) is called *balanced* if each variable occurs exactly twice in $w \doteq w'$, once on each side. The length $l(w \doteq w')$ of an identity $w \doteq w'$ is the sum of the lengths of w and w'.

Let w, w_1 be non-empty words on an algebra of quasigroups (Q, F), and let w_1 be a subword of w. We define $Z(w, w_1)$ as the set of non-negative integers as follows:

(i) $Z(w, w) = \{0\}$ for any word w,

(ii) $n \in Z(w, w_1)$ for $w \neq w_1$ iff there exists a word w_2 of length $n + 1$ such that either $w_1 \,.\, w_2$ or $w_2 \,.\, w_1$ is a subword of w.

Let f be an n-ary operation on a set Q and let $\alpha_1, \alpha_2, \ldots, \alpha_{n+1}$ be permutations of Q. Then the n-ary operation $\alpha_{n+1} f(\alpha_1 x_1, \ldots, \alpha_n x_n)$ is called an isotope of f; the isotope will be denoted by $f^{(\alpha_1, \ldots, \alpha_{n+1})}$. An algebra (Q, F) is called an *isotope of an algebra* (Q, F') if every $f \in F$ is an isotope of some $f' \in F'$ and conversely. It is known that every isotope of a quasigroup is a quasigroup. An isotope of a group is called a transitive quasigroup.

A property P of a quasigroup $(Q, \cdot)$ is said to be *universal* if every isotope of $(Q, \cdot)$ has property P. It is known that the transitivity of quasigroups is a universal property of quasigroups.

Let (Q, F) be an algebra of quasigroups, W the set of all words on the algebra (Q, F) and let (W, F) be the algebra of words on (Q, F). Let $x, y, \ldots, z$ be variables. A map δ: $W \to W$, $w \mapsto \delta w$, where δw is the word that we get from w by deleting all variables except $x, y, \ldots, z$, all superfluous operations and all superfluous brackets (in this order) is said to be a deleting map with invariant variables $x, y, \ldots, z$ (briefly a deleting map). Obviously, each deleting map is an endomorphism.

Let (W, F), (W', F') be the algebras of words on algebras of quasigroups. A map $\omega\colon W \to W'$, $w \mapsto \omega w$, where ωw is the word that we get from w if each operation symbol in w is replaced by an operation symbol in F' (equal operation symbols in w are not necessarily replaced by the same symbol) is called a *change operation map*; if each operation in F' is an isotope of an operation in F, we say that ω is a *change isotopy operation map.*

Let $w \simeq w'$ be an identity on an algebra of quasigroups (Q, F) and let P be a property of a quasigroup $(Q, \cdot)$, where the operation $(\cdot)$ occurs in $w \simeq w'$; we say that P is an invariant (isotopy invariant) of $w \simeq w'$ if for every change operation (resp. change isotopy operation) map ω there exists an operation $\circ$ occuring in $\omega w \simeq \omega w'$ such that $(Q, \circ)$ has the property P.

1.1. Lemma. *Let $w \simeq w'$ be a balanced identity of length $\leqq 6$ on an algebra of quasigroups (Q, F). Then $w \simeq w'$ is equivalent to at least one of the following identities*

(I) $x \,.\, (y \,\square\, z) = x \circ (y \,\triangledown\, z)$

(II) $x \,.\, (y \,\square\, z) = (x \circ y) \,\triangledown\, z$

given on the algebra $(Q, \Sigma F)$.

Proof. Let $V(w) = V(w') = \{x, y, z\}$. First, let $Z(w, x) = Z(w', x) = 1$; then $w = x \,.\, (y \,\square\, z)$ and $w' = x \circ (y \,\triangledown\, z)$ for convenient operations $\cdot, \square, \circ, \triangledown$ in ΣF (if, for example, $w = (z \blacksquare y) \otimes x$ we put $(\cdot)$ to be the dual of $\otimes$, $\square$ the dual of $\blacksquare$). Now, let $Z(w, x) = Z(w', z) = 1$ i.e. there exists no $t \in V(w)$ such that $Z(w, t) = Z(w', t) = 1$; then obviously $w = x \,.\, (y \,\square\, z)$, $w' = (x \circ y) \,\triangledown\, z$ for convenient operations in ΣF. This completes the proof.

Identity (II) is called the *general associative law* (see [3, p. 76]).

1.2. Lemma. *A balanced identity $w \simeq w'$ of length $\leqq 6$ on an algebra of quasigroups (Q, F) is equivalent to the general associative law iff there exists $t \in V(w \simeq w')$ such that $Z(w, t) + Z(w', t) = 1$.*

Proof. Evident.

1.3. Theorem (about four quasigroups, see [1], [3]). *An algebra of quasigroups $(Q, \cdot, \circ, \square, \triangledown)$ that satisfies the identity (II) is an algebra of transitive quasigroups all isotopic to the same group.*

Proof. From (II) it follows that

$$b \circ y = a \circ x \Leftrightarrow L_b L_y^{\square} = L_a L_x^{\square} \Leftrightarrow L_a^{-1} L_b = L_x^{\square}(L_y^{\square})^{-1}\,,$$

$$d \circ y = c \circ x \Leftrightarrow L_d^{\square} L_y^{\square} = L_c L_x^{\square} \Leftrightarrow L_c^{-1} L_d = L_x^{\square}(L_y^{\square})^{-1}\,,$$

therefore

$$b \circ y = a \circ x \quad \text{and} \quad c \circ x = d \circ y \Leftrightarrow L_a^{-1} L_b = L_c^{-1} L_d\,.$$

The simultaneous equations $b \circ y = a \circ x$, $c \circ x = d \circ y$ have a solution for arbitrary three elements of the set $\{a, b, c, d\}$, so according to Theorem 2.1 in [5], $(Q, \cdot)$ is a transitive quasigroup. For the rest of the proof see [3, p. 77].

1.4. Corollary. *The transitivity of a quasigroup is an invariant of the general associative law.*

2. GENERALIZATIONS OF A THEOREM OF BELOUSOV

2.1. Lemma. *Let w be a non-empty word on an algebra of quasigroups (Q, F), let δ be a deleting and ϱ a retraction map, both with invariant variables $x, y, \dots z$. Then there exists a change isotopy operation map ω such that $\varrho w = \omega\delta w$.*

Proof. We shall proceed by induction on the length of w. Let $l(w) = 1$; if δw is empty then $\varrho w = a \in Q$ and we put $\omega\delta w = a$ (an isotope of a 0-ary operation is a 0-ary operation); if δw is non-empty, say $w = x$, then $\delta w = x$ and we put $\omega = 1$. Further, assume that the theorem is valid for all words of length $<n$ and let $1 < l(w) = n$. Then $w = w_1 \,.\, w_2$, where $l(w_1) < n$, $l(w_2) < n$, therefore there exists a change isotopy operation map ω such that $\varrho w_1 = \omega\delta w_1$ and $\varrho w_2 = \omega\delta w_2$. Let δw_1, δw_2 be non-empty words; since $\varrho(w_1 \,.\, w_2) = \varrho w_1 \,.\, \varrho w_2$ we define $\omega(\delta w_1 \,.\, \delta w_2) = \omega\delta w_1 \,.\, \omega\delta w_2$ (i.e. the operation $(\cdot)$ is not changed). Since δ is an endomorphism, $\varrho w = \varrho w_1 \,.\, \varrho w_2 = \omega\delta w_1 \,.\, \omega\delta w_2 = \omega(\delta w_1 \,.\, \delta w_2) = \omega\delta(w_1 \,.\, w_2) = \omega\delta w$. Now suppose that δw_1 is empty and δw_2 is non-empty word. Then there exists a permutation α of Q such that $\varrho w = \alpha\varrho w_2$. Since $l(w_2) < n$, there exists a change isotopy operation map ω such that $\varrho w_2 = \omega\delta w_2$, where $V(\omega\delta w_2) \subset \{x, y, \dots, z\}$ (therefore $\sigma\omega\delta w_2 = \omega\delta w_2$ for every retraction map σ with invariant variables $x, y, \dots, z$). If $\omega\delta w_2 = w_3 \circ w_4$ for some non-empty words w_3, w_4 then we put $\alpha(w_3 \circ w_4) = w_3 \nabla w_4$, thus $\varrho w = \alpha\varrho w_2 = \alpha\omega\delta w_2 = \alpha(w_3 \circ w_4) = w_3 \triangledown w_4$ and that is the above case. If $\delta w_2 = x$, then by the induction hypothesis $\varrho w_2 = \omega'\delta w_2 = \beta x$, where ω' is a change isotopy operation map and β is a permutation of Q; now we put $\omega(\varepsilon) = \alpha\beta\varepsilon$, where ε is the identity map on Q. We have $\varrho w = \alpha\varrho w_2 = \alpha\beta x = \alpha\beta\varepsilon x = \omega(\varepsilon x) = \omega\delta w_2 = \omega\delta w$.

2.2. Theorem. *Let an algebra of quasigroups (Q, F) satisfy an identity $w \simeq w'$ and let δ be a deleting map. Let a universal property P of a quasigroup be an isotopy invariant of $\delta w \simeq \delta w'$. Then there exists $(\cdot)$ in F such that $(Q, \cdot)$ has the property P.*

Proof. Let ϱ be a retraction map with the same invariant variables as δ. By Lemma 2.1, there exists a change isotopy operation map ω such that $\varrho w = \omega\delta w$ and $\varrho w' = \omega\delta w'$. Since $w = w'$, $\varrho w = \varrho w'$ and hence $\omega\delta w = \omega\delta w'$. Therefore by the assumption of the theorem there exists an operation $\circ$ in $\omega\delta w \simeq \omega\delta w'$ such that $(Q, \circ)$ has the universal property P. Because any operation occuring in $\delta w \simeq \delta w'$ is an isotope of an operation in $\omega\delta w \simeq \omega\delta w'$ and any operation occuring in $\delta w \simeq \delta w'$ is in F, there exists $(\cdot)$ in F that is an isotope of $\circ$. Since P is a universal property, $(Q, \cdot)$ has the property P.

2.3. Corollary. *Let an algebra of quasigroups (Q, F) satisfy an identity $w \doteq w'$ and let δ be a deleting map. If $\delta w \doteq \delta w'$ is equivalent to the general associative law on $(Q, \Sigma F)$ then there exists $\circ$ in F such that $(Q, \circ)$ is a transitive quasigroup.*

Proof. Follows from the statement that the transitivity of a quasigroup is a universal property, from Corollary 1.4 and Theorem 2.2.

The identities $w \doteq w'$ that satisfy the conditions of Corollary 2.3 include, for example, the general medial law (see [3, p. 76]).

2.4. Theorem (of Belousov). *Let $w \doteq w'$ be a balanced identity on a quasigroup $(Q, \cdot)$, let $x \,.\, y$ be a subword of w' and let neither $x \,.\, y$ nor $y \,.\, x$ be a subword of w. Then $(Q, \cdot)$ is a transitive quasigroup.*

Proof. Since $x \,.\, y$, $y \,.\, x$ are not subwords of w, there exist at least three variables, say x, y, z, in w. Let δ be the deleting map with invariant variables x, y, z. Then obviously $Z(\delta w, z) = 0$ and $Z(\delta w', z) = 1$ so that $Z(\delta w, z) + Z(\delta w', z) = 1$. Evidently $\delta w \doteq \delta w'$ is a balanced identity, therefore by Lemma 1.2, the identity is equivalent to the general associative law.

3. SOME CLASSES OF TRANSITIVE QUASIGROUPS

3.1. Theorem. *Let $w = w'$ be an identity on an algebra of quasigroups (Q, F) such that*

(1) *$V(w \doteq w') = \{x, y, z\}$ and each variable occurs exactly twice in $w \doteq w'$,*
(2) *$w \doteq w'$ is not of type $x \circ w_1 = x$ or $w_1 \circ x = x$,*
(3) *if a word w_1 of length 2 is a subword of $w \doteq w'$ then $V(w_1)$ consists of two distinct variables,*
(4) *if a word w_1 of length 3 is a subword of $w \doteq w'$ then $V(w_1) = \{x, y, z\}$,*
(5) *for each $t \in \{x, y, z\}$, $\{Z(w, t), Z(w', t)\} \cap \{\{1, 2\}, \{2, 3\}\} = \emptyset$,*
(6) *if words w_1, w_2 of length 2 occur in $w \doteq w'$ as subwords then $V(w_1) \neq V(w_2)$.*

Then

(i) *for every change operation map ω the identity $\omega w \doteq \omega w'$ satisfies all conditions (1)–(6),*
(ii) *there exists a balanced identity of length 6 on $(Q, \Sigma F)$ equivalent to the general associative law,*
(iii) *there exists a group $(Q, \circ)$ such that $(Q, \cdot)$ is an isotope of $(Q, \circ)$ for every operation $(\cdot)$ in $w \doteq w'$.*

Proof. (i) is easy. (ii) Without loss of generality assume $l(w) \leqq 3$. Obviously there exists a variable, say x, that occurs exactly once on each side of $w \doteq w'$. Therefore, there exist $A, B, C, D \in \underset{\circ}{T}^F$ and subwords r, s, t, v of w with variables y, z such that if $w \doteq w'$ is rewritten with translations of (Q, F) we get

$$\text{(a) } A_r B_s x = x \quad \text{or} \quad \text{(b) } A_r B_s C_t x = x \quad \text{or} \quad \text{(c) } A_r B_s C_t D_v x = x$$

($A_r x = x$ contradicts (2)). In the case (a), we have $l(r) = l(s) = 2$, hence $r = y \,.\, z$, $s = y \circ z$, but this contradicts (6). If (b) holds then $\{r, s, t\} = \{y, z, y \,.\, z\}$; first we put $r = y$, $s = z$, $t = y \,.\, z$ then $C_{y.z}x = B_z^{-1}A_y^{-1}x$ is equivalent to (ii). If $r = y \,.\, z$, $s = z$, $t = y$ then $B_z C_y x = A_{y.z}^{-1}x$ is equivalent to (ii). Finally, if $r = y$, $s = y \,.\, z$, $t = z$ then put $y \,.\, z = u$ i.e. $y = u/z$ so that $B_u C_z x = A_{u/z}^{-1}x$ is equivalent to (ii). (iii) follows from Theorem 1.3.

3.2. Corollary. *A balanced identity $w \simeq w'$ of length $\leqq 6$ on an algebra of quasigroups satisfies all conditions (1)–(6) of Theorem 3.1 iff there exists $t \in V(w \simeq w')$ such that $Z(w, t) + Z(w', t) = 1$.*

Proof. Easy.

The identities that satisfy the conditions (1)–(6) of Theorem 3.1 and that are not balanced include, for example, the following identities (see [3, p. 59]): $yx \,.\, xz = yz$, $yx \,.\, zx = yz$, $xz \,.\, xy = yz$, $x \,.\, z(yx) = yz$, $x(yz \,.\, yx) = z$, $(y \,\square\, x) \,.\, (z \circ x) = y \,\triangle\, z$, $(x \,\square\, y) \,.\, (y \circ z) = x \,\triangledown\, z$.

3.3. Theorem. *Let $w \simeq w'$ be an identity on an algebra of quasigroups (Q, F) such that the conditions (1), (2) and (3) of Theorem 3.1 hold. Then there exists a balanced identity on $(Q, \Sigma F)$ equivalent to $w \simeq w'$.*

Proof. (i) Let a word w_1 of length 3 be a subword of w and let $V(w_1) = \{x, y\}$, i.e. (4) of Theorem 3.1 be not valid. Then $w_1 = x \circ (x \,\square\, y)$ for some convenient operations $\circ$, $\square$ in F. If w_1 is expressed from $w \simeq w'$ then we get $w_1 \simeq w_2$, where $w_2 = z \,.\, (z \,\triangledown\, y)$ for some $\cdot$, $\triangledown \in \Sigma F$. Further $w_1 \simeq w_2$ is rewritten with tranlations, so $L_x^{\circ} L_x^{\square} y = L_z L_z^{\triangledown} y$ whence $L_z^{-1} L_{xy}^{\circ} = L_z^{\triangledown}(L_x^{\square})^{-1} y$ is equivalent to (I). (ii) Let $Z(w, x) \in \{\{1, 2\}, \{2, 3\}\}$ i.e. (5) of Theorem 3.1 is not valid. If $Z(w, x) = \{1, 2\}$ then $w \simeq w'$ is equivalent to $x \,.\, (x \circ (y \,\square\, z)) = y \,\triangledown\, z$ and $x \circ (y \,\square\, z) = x \setminus (y \,\triangle\, z)$. If $Z(w, x) = \{2, 3\}$ then $w \simeq w'$ is equivalent to $x \,.\, (x \circ (y \,\square\, (y \,\triangledown\, z))) = z$ whence $Z(w, y) = \{0, 1\}$, that is (i). (iii) Let $x \circ y$, $x \,\square\, y$ be subwords of $w \simeq w'$. Then $w \simeq w'$ is equivalent to $(z \,\triangledown\, (x \circ y)) \,.\, (x \,\square\, y) = z$ as well as $z \,\triangledown\, (x \circ y) = z/(x \,\square\, y)$. Finally, let (i)–(iii) be not valid. Then the statements (1)–(6) of Theorem 3.1 hold and therefore we can use the theorem.

3.4. Theorem. *Let $W \simeq W'$ be an identity on an algebra of quasigroups (Q, F) and let there exist variables $x, y, z \in V(W \simeq W')$ such that if δ is the deleting map with invariant variables x, y, z and $\delta W = w$, $\delta W' = w'$ then $w \simeq w'$ is an identity which satisfies conditions (1)–(6) of Theorem 3.1. Then there exists a group $(Q, \circ)$ such that for every operation $(\cdot)$ in $w \simeq w'$ $(Q, \cdot)$ is an isotope of $(Q, \circ)$.*

Proof. Follows from Corollary 2.3 and Theorem 3.1.

3.5. Theorem. *Let w be a word on an algebra of quasigroups (Q, F), $V(w) = \{x, y\}$, $A, B \in T^F$ and let δ be the deleting map with invariant variables x, y, z. If $W = W'$ is an identity on (Q, F) such that $\delta W \simeq \delta W'$ is at least one of the* *identities*

(7) $$A_z(B_z x \,.\, y) \doteq w$$

(8) $$A_z(x \,.\, B_z y) \doteq w$$

then there exists a group $(Q, +)$ *such that for every operation* $(\cdot)$ *in* $\delta W \doteq \delta W'$, $(Q, \cdot)$ *is an isotope of* $(Q, +)$.

Proof. Let us denote $w = x \circ y$, $z \triangledown x = B_z x$, $A_z^{-1} t = z \,\square\, t$ for all x, y, t. Then from (7) we have

(i) $$(z \triangledown x) \,.\, y \doteq z \,\square\, (x \circ y) \,.$$

Since $(\circ) = (\cdot)^{(B_z, 1, A_z)}$, $(Q, \circ)$ is a quasigroup. Thus (i) is the general associative law. The rest of the proof is similar.

Among identities of type (7) belongs the identity $z(xz \,.\, y) = x \,.\, xy$ (see [4]). From theorem 3.5 directly follows that a quasigroup which satisfies this identity is a transitive quasigroup.

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