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ON SUPREMA OF METRIZABLE VECTOR TOPOLOGIES WITH TRIVIAL DUAL

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INTRODUCTION

Following [13] a vector topology τ on a vector space E will be called *dual-less* if (E, τ) has no non-trivial continuous linear functionals; in this case we shall say that (E, τ) is a *dual-less space*. This is the case when all absorbing and convex [semi-convex] subsets of E are everywhere dense. Such a topology will be called (after Peck and Porta) dual-less of type e [se].

In the present paper we return to the following problems investigated in [11], [12], [13]:

(a) *Which vector topologies can be expressed as suprema of dual-less topologies?*

(b) *Which vector topologies are restrictions of dual-less topologies on a larger space?*

(c) *Which vector topologies admit weaker dual-less topologies?*

In [13], Theorem C, Peck and Porta proved: The topology of the product space $E \times \ldots \times E$ (n times, $n \geqq 2$) of an infinite dimensional separable normed space E is the supremum of $n + 1$ dual-less topologies. Hence, in particular, the norm topology on each of the following Banach spaces: $L^p[0, 1]$, l^p $(1 \leqq p < \infty)$, $C[0, 1]$, c_0, is the supremum of three dual-less topologies. Unfortunately, the construction carried out by the authors does not ensure Hausdorff's property of the dual-less topologies obtained.

We shall say that a metrizable non dual-less topological vector space (tvs) $E = (E, \tau)$ has the *property* (i_p), $p \geqq 2$, if τ is the supremum of p metrizable dual-less topologies; replacing in (i_p) "metrizable" by "locally bounded and Hausdorff" we obtain the *property* (j_p).

Our main results concerning (a) are the following theorems.

Theorem 0. *Let E be an infinite dimensional separable [and locally bounded] F-space such that its topological dual E' has an equicontinuous and total sequence* *over E. Then the product space $E \times E$ has the property* (i_3) $[(j_3)]$ *and E admits a strictly finer metrizable [and locally bounded] separable Baire topology under which E has the property* (i_3) $[(j_3)]$.

Theorem 00. *Let (E, τ) be the product space of two separable [and locally bounded] F-spaces E_1 and E_2 with* $\dim E_1 = \dim E_2 = \infty$. *If every E_k', $k = 1, 2$, has an equicontinuous and total sequence over E_k, (E, τ) has the property* (i_4) $[(j_4)]$.

The proofs of the above theorems will be based on some ideas used in [13] combined with recent results concerning summable sequences in tvs.

Clearly, Theorems 0 and 00 apply when E is an infinite dimensional separable Banach space; as concerns non locally convex spaces, Theorem 0 shows in particular that the topology of every sequence space l^p $(0 < p < 1)$ is the supremum of three locally bounded Hausdorff dual-less topologies.

We indicate also a number of spaces to which Theorems 0,00 apply; among others, using Corollary 3.6 of [4], in every non-minimal separable F-space E we find a pair of proper quasi-complements G_1 and G_2 to which Theorem 0 applies; if E is non locally convex but nearly convex, i.e. E' is point-separating, G_1 and G_2 can be chosen so that the quotients E/G_k, $k = 1, 2$, are dual-less. This fact partially extends Klee's result of [8] concerning the existence of metrizable spaces E which are algebraic direct sums of closed subspaces G with dual-less quotients E/G. Recall that two closed subspaces G_1 and G_2 of a tvs E are quasi-complements if $G_1 \cap G_2 = 0$ and $G_1 + G_2$ is dense in E.

We also prove that every infinite dimensional F-space, i.e. a metrizable and complete tvs, admits a strictly finer vector topology different from the finest one which is the supremum of three Hausdorff dual-less topologies of type e. This partially solves the problem raised by Peck and Porta in [13], Section 3.

Results concerning the problem of finding a weaker non locally convex [and dual-less] topology on a given non dual-less tvs complete this paper; we also list some open problems.

All the tvs which will appear are supposed to be infinite dimensional and Hausdorff. By a subspace of a tvs (E, τ) we mean a vector subspace G endowed with the induced topology; the resulting tvs will be written as $(G, \tau \mid G)$. A tvs (E, τ) is dominated [strictly dominated] by an F-space if there exists on E a finer [strictly finer] vector topology ϑ such that (E, ϑ) is an F-space.

A sequence (y_i) in E is called *bounded multiplier summable* (BMS) provided $\sum_{i=1}^{\infty} t_i y_i$ converges in E for all $(t_i) \in m := l_\infty$.

Following [9] a sequence (y_i) in E is called *(linearly) m-independent* if $(t_i) \in m$ and $\sum_{i=1}^{\infty} t_i y_i = 0$ imply $(t_i) = 0$. According to [2], Lemma 2, for every linearly independent sequence (y_i) in E there exists a scalar sequence (d_i), $d_i > 0$, such that $(d_i y_i)$ is m-independent. Hence, if E has a linearly independent (BMS)-sequence (y_i), we may replace (y_i) without changing its linear hull by a new one which is (BMS) and m-independent.

Let G be a closed subspace of a tvs E and $Q: E \to E/G$ the quotient map. Following [4] we shall say that a sequence (y_i) is m-independent of G if $(Q(y_i))$ is m-independent in E/G; clearly, then (y_i) is m-independent in E.

The following fact will be used in the sequel, cf. [4], Proposition 2.1.

(A) *Let G be a closed subspace of a tvs E such that E/G is metrizable separable and infinite dimensional. Let W be a subspace of E such that $W \cap G = 0$ and $W + G$ is dense in E. Then W contains a sequence (y_i) which is m-independent of G and* $\operatorname{lin}(y_i) + G$ *is dense in E.*

A tvs E is said to have the *property* (K) [1] if every sequence (y_i) in E with $y_i \to 0$ has a subsequence (x_i) such that $\sum_{i=1}^{\infty} x_i$ converges in E. In [1], Theorem 2, it is proved that:

(B) *If E is metrizable and has the property* (K), *E is a Baire space.*

We shall need also the following fact, cf. [10], Theorem 4.

(C) *Every F-space of dimension $c = 2^{\aleph_0}$ is the algebraic direct sum of two dense subspaces G_1 and G_2 with the property* (K) *such that $G_1 \times G_2$ has the property* (K) *as well.*

Note that for every separable infinite dimensional F-space E we have $\dim E = c$, [9], Corollary 2. Finally, a vector topology τ on a vector space E will be called a *Baire topology* if (E, τ) is a Baire space, i.e., is of Baire's second category.

RESULTS

We start with the following

Lemma 1. *Let (E, τ) be a separable [and locally bounded] dual-less F-space and G its closed subspace such that G' has an equicontinuous and total sequence over G. Then $G \times G$ has the property* (i_3) $[(\mathrm{j}_3)]$ *and G admits a strictly finer metrizable [and locally bounded] separable Baire topology under which G has the property* (i_3) $[(\mathrm{j}_3)]$.

Proof. By (A) we find in E a (BMS)-sequence (y_i) which is m-independent of G, such that $G + \operatorname{lin}(y_i)$ is dense in E. Using a construction from [14], p. 154, [4], p. 380–381, we find a biorthogonal system (x_i), (f_i) with $(x_i) \subset G$, (f_i) equicontinuous and total over G. Define a compact injective linear map P of G into E by putting $P(x) = \sum_{i=1}^{\infty} f_i(x)\, y_i$; in the sequel we shall call P (after Drewnowski [4]) the compact map determined by the sequences (f_i) and (y_i). Since (y_i) is m-independent of G, $G \cap P(G) = 0$; observe also that $G + P(G)$ is dense in E. By (B) and (C) we find in G two dense Baire subspaces G_1 and G_2 such that $G = G_1 + G_2$ (algebraically) and the topology $\gamma = \tau \mid G_1 \times \tau \mid G_2$ is a Baire topology. Define two continuous injective linear maps $T_k: (G_1 \times G_2, \gamma) \to (E, \tau)$, $k = 1, 2$, by putting

$$T_1(x_1, x_2) = x_1 + P(x_2)\,, \quad T_2(x_1, x_2) = x_2 + P(x_1)\,.$$

Clearly $G_1 + P(G_2)$ and $G_2 + P(G_1)$ are dense in E. Hence the inverse topologies

$\vartheta_k := T_k^{-1}(\tau)$ are metrizable dual-less [and locally bounded] and weaker than γ. Now consider a continuous injective linear map $L: (G_1 \times G_2, \gamma) \to (G_1 \times G_2, \vartheta_1)$ defined by $L(x_1, x_2) := (x_1, -x_2)$. Put $\vartheta_3 := L^{-1}(\vartheta_1)$. We prove $\gamma = \sup(\vartheta_1, \vartheta_2, \vartheta_3)$. Let $x_n := (x_n^1, x_n^2) \to 0$ for $\sup(\vartheta_1, \vartheta_2, \vartheta_3)$. Hence $L(x_n) \to 0$ for ϑ_1, and then $(x_n^1, 0) = 2^{-1}(x_n + L(x_n)) \to 0$ for ϑ_1. Therefore $T_1(x_n^1, 0) = x_n^1 \to 0$ for $\tau \mid G_1$, and hence we obtain $(x_n^1, 0) \to 0$ for $\sup(\vartheta_1, \vartheta_2, \vartheta_3)$. Since we have $(0, x_n^2) \to 0$ for ϑ_2, $T_2(0, x_n^2) = x_n^2 \to 0$ for $\tau \mid G_2$, so $x_n \to 0$ for γ.

Finally, since the map $(x_1, x_2) \mapsto x_1 + x_2$, which maps $G_1 \times G_2$ onto G, is continuous and injective but not open, G admits a strictly finer vector topology as claimed.

The remaining case is obtained similarly: Define a continuous and injective linear map $T_1: (G \times G, \tau \mid G \times \tau \mid G) \to (E, \tau)$ by putting $T_1(x_1, x_2) := x_1 + P(x_2)$. Let $\vartheta_1 := T_1^{-1}(\tau)$. Next, consider two maps T_2 and T_3 of $G \times G$ onto $G \times G$ defined by $T_2(x_1, x_2) := (x_1, -x_2)$, $T_3(x_1, x_2) := (x_2, x_1)$. Putting $\vartheta_k := T_k^{-1}(\vartheta_1)$, $k = 2, 3$, we obtain on $G \times G$ the desired topologies such that $\tau \mid G \times \tau \mid G = \sup(\vartheta_1, \vartheta_2, \vartheta_3)$.

Proof of Theorem 0. Let E be a vector space. We shall say that a function $f: [0, 1] \to E$ is simple if there exist a finite number of disjoint subsets $A_1, A_2, \ldots, A_n$ of $[0, 1]$ whose union is $[0, 1]$ and $x_1, x_2, \ldots, x_n \in E$ such that $f(t) = \sum_{i=1}^n x_i \chi_{A_i}(t)$, where χ_A denotes the characteristic function of the set A. Let $L(E)$ be the set of all simple functions from $[0, 1]$ into E. Clearly the pointwise operations induce a vector structure on $L(E)$. Assume E is a separable locally bounded space whose topology is generated by a q-norm $\| \ \|$ $(0 < q \leqq 1)$. Fix $0 < p < 1$ and put

$$|||f|||_p := \int_0^1 \|f(t)\|^p \, dt = \sum_{i=1}^n \|x_i\|^p \mu(A_i),$$

where $f \in L(E)$ and μ denotes the Lebesgue measure on $[0, 1]$. As is easily seen, the space $L(E)$ equipped with the functional $||| \ |||_p$ is a pq-normed dual-less separable space of type e, so its completion is a space of the same type. Since the map $x \mapsto f_x$, where $f_x(t) := x$, $t \in [0, 1]$, is an isomorphism of E into $L(E)$, Lemma 1 applies to conclude the first part of the proof.

If E is not necessarily locally bounded we consider on $L(E)$ the topology of convergence in measure investigated in the proof of Theorem 1.1, [12], and apply Lemma 1.

Clearly every separable Banach space satisfies the assumptions of Theorem 0. The simplest non locally convex spaces to which Theorem 0 applies are the spaces of sequences l^p $(0 < p < 1)$. Since l^p is isomorphic to its own square and is continuously embedded into a dense subspace of l^1, l^p has the property (j_3).

Proof of Theorem 00. Let $(x_i^k), (f_i^k)$, $k = 1, 2$, be two biorthogonal systems such that $(x_i^k) \subset E_k$ and (f_i^k) is equicontinuous and total over E_k. For every $k = 1, 2$ let T_k be an isomorphism of E_k into the completion (H, ϑ) of $L(E)$ (constructed in the previous proof). By (A) we find in H an m-independent of $T_k(E_k)$ (BMS)-sequence

(y_i^k) such that $\operatorname{lin}(y_i^k) + T_k(E_k)$ is dense in H. We construct two injective compact linear maps $P_1: E_1 \to H$, $P_2: E_2 \to H$ determined by the sequences $(f_i^1), (y_i^2)$ and $(f_i^2), (y_i^1)$, respectively. Observe that $P_k(E_k) \cap T_r(E_r) = 0$ and $P_k(E_k) + T_r(E_r)$ is dense in H for evety $k, r = 1, 2$, $k \neq r$. Now we define injective and continuous linear maps

$$U_k(x_1, x_2) := T_1(x_1) + (-1)^k P_2(x_2) \quad \text{for} \quad k = 1, 2 \quad \text{and}$$
$$U_k(x_1, x_2) := P_1(x_1) + (-1)^k T_2(x_2) \quad \text{for} \quad k = 3, 4\,.$$

Put $\tau_k := U_k^{-1}(\vartheta)$ for $1 \leqq k \leqq 4$. It is not hard to prove that $\tau = \sup(\tau_k\colon 1 \leqq \leqq k \leqq 4)$, and the proof is complete.

Recall that a tvs E is *minimal* if E does not admit a strictly weaker Hausdorff vector topology, and *non-minimal* otherwise. In view of [5], Theorem 3.3, an F-space E is non-minimal if and only if E has a *strongly regular M-basic sequence* (y_i), i.e. there exists a sequence (f_i) biorthogonal to (y_i), equicontinuous and total over the closed linear hull $[(y_i)]$ of (y_i). Let E be a non-minimal separable F-space. In [4], Corollary 3.6, Drewnowski proved that E has a pair of isomorphic proper quasi-complements G_1 and G_2, where $G_1 := [(y_{2i})]$.

Hence we obtain

Corollary 2. *Every non-minimal separable F-space E contains a pair of isomorphic proper quasi-complements G_1 and G_2 to which Theorem* 0 *applies. Moreover, if E is non locally convex but nearly convex, G_1 and G_2 can be chosen so that E/G_k, $k = 1, 2$, are dual-less.*

The last assertion of Corollary 2 will be obvious when we use Theorem 4.1 of [4] and compare the proofs of Theorem 3.3 of [4] and Theorem 1 of [6].

Using Theorem 00 and Corollary 2 we obtain

Corollary 3. *Every separable non-minimal F-space has a dense subspace which is strictly dominated by a separable F-space whose topology is the supremum of four metrizable dual-less topologies.*

Corollary 4. *Let E and G be two separable* [*and locally bounded*] *non locally convex but nearly convex F-spaces. Then the product $E \times G$ has a closed subspace H with the property* (i_4) $[(\mathrm{j}_4)]$, *such that $(E \times G)/H$ is dual-less.*

In [11], Theorem 3.3, it is proved that every separable normed space admits a weaker dual-less topology. We prove a stronger result.

Proposition 5. *Let E be a metrizable tvs such that the topological dual of the completion $\tilde{E}$ of E has an equicontinuous and total sequence over $\tilde{E}$. Then E admits a strictly weaker locally bounded Hausdorff dual-less topology.*

Proof. By the assumption we find a biorthogonal system $(x_i), (f_i)$; $(x_i) \subset \tilde{E}$, (f_i) is equicontinuous and total over $\tilde{E}$. Fix $0 < p < 1$ and consider the locally bounded separable dual-less F-space $H := L^p[0, 1]$. Choose in H an m-independent (BMS)-sequence (y_i) such that $\operatorname{lin}(y_i)$ is dense in H; this is possible by (A). Define a compact injective linear map P of $\tilde{E}$ into H determined by (f_i) and (y_i). Since $P(\tilde{E})$ is dense in H, the inverse topology under P restricted to E is as required.

Corollary 6. *Every non-minimal F-space E has a closed infinite codimensional subspace which admits a strictly weaker locally bounded Hausdorff dual-less topology.*

Proof. Take in E a strongly regular M-basic sequence (x_i) and apply Proposition 5 to the space $G := [(x_{2i})]$.

Corollary 7. *Every non-minimal [and locally bounded] F-space (E, τ) admits a strictly weaker non locally convex metrizable [and locally bounded] vector topology.*

Proof. By Corollary 6 the space E has a closed subspace G which admits a strictly weaker locally bounded Hausdorff dual-less topology ϑ. Taking the infimum topology γ of ϑ and τ, i.e. the strongest vector topology among the vector topologies ξ on E such that $\xi \leqq \tau$ and $\xi \mid G \leqq \vartheta$, we find on the space E a topology as required.

We do not know whether the topology γ can always be chosen to be dual-less. Nonetheless, we are able to prove the following fact:

Corollary 8. *Every separable non locally convex but nearly convex F-space (E, τ) admits a weaker metrizable dual-less topology ξ and contains a proper ξ-closed subspace G such that the induced topology $\xi \mid G$ is dual-less and $\xi/G = \tau/G$.*

Proof. In view of Corollary 2 and Proposition 5 we find in E a proper closed subspace G such that τ/G is dual-less and G admits a strictly weaker metrizable dual-less topology γ. Hence, the topology α, being the infimum topology of γ and τ, is metrizable, strictly weaker than τ, and $\alpha \mid G = \gamma$; clearly G is α-closed. Denote by ξ the initial topology on E with respect to the identity map $E \to (E, \alpha)$ and the quotient map $E \to (E/G, \tau/G)$. As is easily seen we obtain that $\alpha \leqq \xi < \tau$, $\gamma = \alpha \mid G = \xi \mid G$, $\tau/G = \xi/G$, and the proof is complete.

Proposition 5 leads to

Corollary 9. *Let (E, τ) be a non-minimal F-space. Then the product space $E \times E$ admits a strictly weaker metrizable non locally convex topology ξ such that $\xi \mid E = \tau$.*

Proof. Let (x_i) be a strongly regular M-basic sequence in E. Put $G := \{(x, x): x \in [(x_{2i})]\}$. Since G is isomorphic to $[(x_{2i})]$, by Proposition 5 we obtain on G a strictly weaker metrizable dual-less topology γ. Define ξ to be the infimum topology of γ and $\tau \times \tau$; it is non locally convex, Hausdorff, and strictly weaker than $\tau \times \tau$. In order to show $\xi \mid E = \tau$, it is enough to apply the proof of Theorem 3.3a of [3].

Remark 10. (a) Using an argument of the same type as above we are able to obtain that if (E, τ) and (F, γ) are two F-spaces which have non-minimal isomorphic closed subspaces, there exists on $E \times F$ a metrizable non locally convex vector topology $\xi < \tau \times \gamma$ such that $\xi \mid E = \tau$ and $\xi \mid F = \gamma$. In particular, we derive that the algebraic sum of two normed subspaces of a tvs need not be locally convex in the relative topology.

(b) Within non separable F-spaces we single out the spaces $l^p(\Gamma)$, $0 < p < \infty$, $c_0(\Gamma)$ (Γ is uncountable), which admit weaker metrizable dual-less topologies. We show only the case of $l^p(\Gamma)$ with $0 < p < 1$; the remaining cases were proved similarly in [13], Theorem 2.6, although the construction presented in [13] does not ensure the metrizability of weaker dual-less topologies. Consider a compact injective linear map P of l^p into $L^p[0, 1]$ with dense range (see the proof of Lemma 1). We apply P to deduce existence of a continuous injective linear map of $l^p(\Gamma, l^p)$ (isomorphic to $l^p(\Gamma)$) into a dual-less F-space $l^p(\Gamma, L^p[0, 1])$ with dense range.

It is known [13], Theorem B2, that the finest vector topology of any uncountably dimensional vector space E is the supremum of three type se dual-less Hausdorff topologies. This fact motivates the following question: Does every F-space admit a finer vector topology different from the finest one which is the supremum of dual-less topologies of type e?

Proposition 3.3 of [13] answers "yes" if E is a separable Hilbert space.

We obtain a stronger result for F-spaces.

Proposition 11. *Let E be a tvs having an m-independent (BMS)-sequence. Then E admits a strictly finer vector topology different from the finest one which is the supremum of three dual-less Hausdorff topologies of type e.*

Proof. Fix a separable Hilbert space G. In [7], Proposition 1, we proved that E contains a subspace H strictly dominated by an isomorphic copy (H, ϑ) of G such that $\operatorname{codim} H \geqq \dim H = c$. Let W be an algebraic complement of H in E ($\dim W \geqq c$) endowed with the finest vector topology γ. Using Peck's and Porta's results mentioned above ([13], Theorem B2, Proposition 3.3) we obtain that $\vartheta \times \gamma$ generates on E a topology as required.

OPEN PROBLEMS

The author has been unable to answer some questions which arose in the course of preparation of the paper.

Problem 1. *Are Theorems* 0 *and* 00 *valid for general (separable) F-spaces?*

Problem 2. *Does every metrizable tvs whose completion is non-minimal admit a strictly weaker metrizable dual-less topology?*

Problem 3. *Let (E, τ) be a non locally convex separable nearly convex F-space and μ the Mackey topology on E, i.e. the topology induced by all convex τ-neighbourhoods of zero. Does E admit a dual-less topology φ such that* $\tau = \sup(\varphi, \mu)$? (Note that the topology μ cannot be replaced by the weak topology asociated with τ.)

Let E be an uncountably dimensional vector space. Is the finest vector topology *on E necessarily the supremum of the finest locally convex topology and a dual-less topology?*

Problem 4. *Does every dual-less space admit a strictly finer dual-less topology?*

We can make only the following remark concerning 4: every tvs (E, τ) which is metrizable [and complete with $\dim E = c$] admits a strictly finer [and Baire] topology ϑ such that ϑ is dual-less if τ is dual-less. Indeed, in view of [10], Theorem 1, E is the algebraic direct sum of the sequence (E_α) of dense subspaces of E; this enables us to obtain on E a topology as claimed. The remaining case is a consequence of (B) and (C) (see Introduction).

On the other hand, every F-space $(E, \| \ \|)$ admits a strictly finer metrizable Baire topology γ which is the supremum of two metrizable and complete vector topologies; and if E is dual-less, $0 < \dim (E, \gamma)' < \infty$. Indeed, choose in E a dense finite co-dimensional Baire subspace G and let H be its algebraic complement endowed with its unique Hausdorff vector topology φ. Let $|||x||| = \inf\{\|x + x\| : y \in H\}$, $x \in G$, then the F-norm $||| \ |||$ generates on G a weaker metrizable and complete vector topology ϑ. To conclude it is enough to put $\gamma := \sup(\tau, \vartheta \oplus \varphi)$, where τ denotes the topology generated by the F-norm $\| \ \|$.

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