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Book review. Fučík, S., Nečas, J., Souček, J., Souček, V.: Spectral Analysis of Nonlinear Operators

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BOOK REVIEWS

S. Fučík, J. Nečas, J. Souček, V. Souček: SPECTRAL ANALYSIS OF NONLINEAR OPERATORS. Lecture Notes in Mathematics No 346, Springer-Verlag, Berlin—Heidelberg—New York a JČSMF, Praha 1974, pp. 287.

The authors deal with two essential problems of nonlinear spectral analysis which have been solved during the last years in a relatively complete form, i.e., results similar to those known for the linear case have been achieved.

The first problem is that of solvability of the operator equation

$$\lambda\, T(u) - S(u) = f\,, \tag{1}$$

where T, S are nonlinear operators mapping a Banach space X into another Banach space Y, λ being a real parameter.

If $X = Y$ is a Hilbert space, T the identical mapping and S a completely continuous linear operator, then the well-known Fredholm alternative holds. The authors prove its generalization to the case of nonlinear $T, S : X \to Y$, where T "behaves similarly to identity" and S is completely continuous (Chap. 2). Under some additional assumptions on the operators T and S (for example, if T, S are positively a-homogeneous with $a > 0$, i.e., $T(tu) = t^a\, T(u)$, $S(tu) = t^a\, S(u)$ for $t > 0$, $u \in X$, and odd, i.e., $T(-u) = -T(u)$, $S(-u) = -S(u)$) it holds: if λ is not an eigenvalue of the couple T, S (i.e., if the equation $\lambda T(u) - S(u) = 0$ has only trivial solution) then for every $f \in Y$ there exists a solution of Eq. (1). Various forms of this Fredholm alternative for nonlinear operators are given in the book (for example, operators asymptotically near to positively a-homogeneous operators are considered instead of positively a-homogeneous ones, and similar).

The other problem is that of the "number of points" of the set of eigenvalues of a couple of nonlinear operators T, S. In connection with it, functionals f, g on a Banach space X are investigated which have Fréchet derivatives f', g'. A number $\gamma = g(u)$ where u is a point of the manifold $M_r(f) = \{u \in X; f(u) = r\}$ ($r > 0$ given number) to which there exists a real λ such that $\lambda f'(u) - g'(u) = 0$ is called a critical level of the functional g with respect to the manifold $M_r(f)$. If the operators $f' = T$, $g' = S$ mapping the space X into the dual space X^* are positively a-homogeneous, then the properties of the set of critical levels can be transferred to the set of eigenvalues of this particular couple T, S. The authors explain the Ljusternik-Schnirelmann theory which asserts that (under certain assumptions on the functionals f, g) there exist at least countably many different critical levels of the functional g with respect to the manifold $M_r(f)$ (Chap. 3). Further, it is shown on the basis of a version of the Morse-Sard Theorem for real analytic functions (Chap. 4) that the set of critical levels is precisely a sequence of positive numbers tending to zero (Chap. 5). Hence it follows that under certain assumptions the eigenvalues of the couple T, S form a sequence of positive numbers tending to zero. The same result is obtained by another method for the special case of ordinary differential equations of the second and fourth orders (Appendix V).

Simultaneously, the authors deal with applications of the abstract results to boundary value problems for both ordinary and partial nonlinear differential equations, to integral and integro-differential equations (Appendices II, III, VI).

The book presents an excellent survey of recent results, methods and problems in the mentioned part of nonlinear analysis in which all four authors have achieved deep results (a considerable part of the book consists of original results of the authors). At the same time, the style of the book is both lucid and comprehensive. The reader is assumed to have only the knowledge of widely known fields of the analysis. Those parts of analysis which are not so currently known and which are necessary for deducing the results of the book are explained from the beginning. For example, the authors give the definition and derive the properties of the Brouwer and Leray-Schauder degree of a mapping (Chap. 1) which is of crucial importance in the proof of the Fredholm alternative for nonlinear operators. (However, a little too big logical gap appears when introducing the Brouwer degree — Lemma 2.3.) Real analytic functions as well as operators in Banach spaces are studied and a special version of the Morse-Sard Theorem is proved (Chap. 4) on which the upper estimate of the number of eigenvalues is based.

New results have been achieved since the book was written in solving some problems indicated there as open (particularly, this concerns the complicated problem of what is the range of the operator $\lambda T - S$ where λ is an eigenvalue). The reader can find these results in several new papers recently published by the authors.

In the conclusion, the readiness with which JČSMF together with the Springer-Verlag published the book should be appreciated.

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