News and Notices

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NEWS AND NOTICES

ZDENĚK FROLÍK AWARDED THE STATE PRIZE

Dr. ZDENĚK FROLÍK, DrSc. has been awarded Klement Gottwald State Prize this year for the constitution and application of general descriptive theory of spaces and sets.

He started his scientific career as graduate student in the Institute of Mathematics of the Charles University, under the guidance of M. Katětov. During that time he sent to the press more than 10 original papers and completed his postgraduate studies within a shortened period of 2 years in 1959. Most of his papers written in the 1960's (from 1961 to 1966 he was employed as research worker in the Institute of Mathematics of the Charles University and later on as research worker in the Institute of Mathematics of the Czechoslovak Academy of Sciences) belong to basic papers in the field concerned and are frequently quoted. Let us recall e.g. papers on topologically complete spaces or on realcompact spaces and their generalizations, the papers "*The topological product of pseudocompact spaces*", Czech. Math. J. *10* (1960), 339—349 concerning the equality $\beta(P \times Q) = \beta P \times \beta Q$ and "*On the topological product of paracompact spaces*", Bull. Acad. Polon. Sci. *8* (1960), 747—750 the essential ideas of which reappeared later in investigations of M-spaces of K. Morita and of p-spaces of A. Arhangelskii. It is worth noting here that some concepts and results introduced by Frolík more than 10 years ago started to be used intensively in the last years (pseudo-m-compactness, z-closed mappings etc.). At about the same time the first Frolík's results on separable descriptive theory of spaces and sets appeared culminating in the paper presented under the title "*A contribution to the descriptive theory of sets and spaces*" at the First Prague Topological Symposium in 1961. The results obtained by Frolík in descriptive theory were in agreement with those achieved after World War II mainly by G. Choquet. They were, howewer, more general and offered the possibility of a further development. Frolík' generalization was motivated not only by the needs of topology but had also an impact on other fields of mathematics (propability theory, mathematical analysis).

The classic descriptive theory in separable metric spaces is, in a sense, based on continuous mappings from the space Σ of irrational numbers; e.g. the analytical spaces in the class of all metric separable spaces are exactly continuous images of the space Σ. Frolík generalized this theory using many-valued mappings. A many-valued mapping $f: P \to Q$ is called *upper semi-continuous* (usco) if the preimages of closed sets are closed, *compact* if the values fx are compact and finally *disjoint upper semi-continuous* (dusco) if usco and the images of distinct points are disjoint. For simplicity we shall limit ourselves to completely regular Hausdorff spaces. Z. Frolík called a space *analytic* (or *Borelian*) if it is an image of Σ under an usco-compact mapping (or dusco-compact mapping). He called the space P *bianalytic* if both P and $\beta P - P$ are analytic spaces (then the Borelian spaces are exactly the simple continuous images of bianalytic spaces). Evidently each analytic space (hence in particular the Borelian and the bianalytic ones) is a Lindelöf space so that restricting the given definitions to metrizable spaces we get separable spaces; in this case it is possible to limit oneself to single-valued continuous mappings and the Borelian and the bianalytic spaces then coincide with separable absolute Baire metrizable spaces. The basic results on metrizable spaces may be applied to a general case; we will mention some of them (recall that the Souslin sets may be defined as images of Σ

under multi-valued mappings with a closed graph — another result of Frolík): (1) *The class of all analytic (Borelian) spaces is closed-hereditary, countably productive and closed under usco-compact (dusco-compact) mappings*; (2) *Analytic spaces are exactly the absolute Souslin spaces*; (3) *If A is an analytic subspace of P, B a Souslin set in P disjoint with A, then there exists a Baire set C such that $A \subset C \subset P - B$ and a continuous mapping f on P into a separable metrizable space such that $f[A] \cap f[B] = \emptyset$.* In addition, for a general case Z. Frolík proved propositions which, applied to the metrizable case, brought so far unknown and sought for results. One of them is e.g. an intrinsic characterization of Borelian spaces. Let us first introduce the following notion: The system of coverings of the space P is called *complete* if any filter in P containing at least one set from each member of the system has an accumulation point. Borelian spaces are then the spaces having a complete countable system $\{\mathscr{M}_n\}_1^\infty$ of disjoint countable coverings with the property: $\bigcap_{n=1}^{\infty} M_n = \bigcap_{n=1}^{\infty} \overline{\bigcap_{k=1}^{n} M_k}$, whenever $M_n \in \mathscr{M}_n$ (this property may be substituted by the following one: the members of $\mathscr{M}_n$ are analytic subspaces of P). In a similar way, the analytic spaces may be characterized as spaces in which a complete sequence of countable coverings exists. These and other resuíts may be found in the above mentioned contribution published in Proc. Prague Top. Symp. 1961.

Later on, Z. Frolík extended and deepened his results. We cannot dicuss here in detail all his achievements in the field of descriptive theory (he has written more than 30 papers concerning with this topic). Let us mention only some of them. In the paper "*On the Souslin graph theorem*", Comment. Math. Univ. Carolinae *9* (1968), 243—249, an analogon of the well-known closed-graph theorem from the theory of Banach spaces was proved: *Let E be a topological linear space inductively generated by homomorphisms from topological linear spaces of 2nd category and F be an analytic locally convex space; if f is a homomorphism on E into F the graph of which is a Souslin set in $E \times F$, then f is continuous.* The paper "*Stone-Weierstrass theorems for $C(X)$ with the sequential topology*", Proc. Amer. Math. Soc. *27* (1971), 486—494, contains the proof of the following analogon of the Stone-Weierstrass theorem: *If P is an analytic space, $\mathscr{A}$ an algebra of continuous functions on P distinguishing points, then $\mathscr{A}$ is dense in $C(P)$ in the topology of pointwise convergence of sequences.* In the paper "*A measurable map with analytic domain and metrizable range is quotient*", Bull. Amer. Math. Soc. *76* (1970), 1112—1117, Luzin's theorem was generalized to the case expressed by the title of the paper.

Till now we have mentioned only the separable descriptive theory. As far as the generalization of descriptive theory from metric (and non-separable spaces) is concerned, the situation is much more complicated, but also here a considerable progress has been made, due mainly to Z. Frolík and to A. H. Stone. In view of a much higher complexity of the subject we shall not give any details; these may be found along with an extensive references in "*A survey of descriptive theory of sets and spaces*", Czech. Mat. J. *20* (1970), 406—467 where all previous results concerning the descriptive theory of sets and spaces have been summarized. Moreover, there are given possibilities of a further generalization if another space than Σ, and hence also a generalized Souslin operation, is used. A number of mathematicians (e.g. C. A. Rogers, J. E. Jayne, R. W. Hansell) are using extensivelly Frolík's results in their work.

Let us also mention other results obtained by Frolík at the same time and having met with at least as vivid a response as the papers in descriptive theory mentioned above. Of a particular importance are his papers on Čech-Stone β-compactification of the space P. The paper "*Fixed points of maps of ED spaces and complete Boolean algebras, and application to homogenity problems*", Proc. Top. Symp. Kanpur 1968, 131—142 may serve almost as a survey paper on these results with references to other papers. Z. Frolík developed a theory of types of ultrafilters and proved their basic properties. Let us give now in brief some applications of this theory: (1) *If P is not pseudocompact, then $\beta P - P$ is not homogeneous* (without continuum hypothesis); (2) *There exist*

spaces P_n, P such that the spaces P_n^n, P^n are countably compact for any n but P_n^{n+1}, P^{ω_0} are not countably compact; (3) *For any homeomorphism f on a compact extremally disconnected space into itself there is a disjoint decomposition $P = \bigcup_{i=1}^{4} X_i$ on clopen sets such that f is the identity on X_1 and $f[X_i] \cap X_i = \emptyset$ for $i = 2, 3, 4$*; (4) *If x is a fixed point of a continuous mapping f on an extremally disconnected space into itself, then any neighbourhood of x contains an f-invariant clopen neighborhood.*

At present Z. Frolík investigates with increasing intention measurable spaces and problems associated with them (which is of course in close connection with descriptive theory). His main results in this field are concerned with the application of methods of uniform spaces to the measure-spaces and vice-versa (in a general case, not only in a separable one) and with the projective limits of presheaves of the measure-spaces; at the 6th Symposium on Mathematical Statistics and on Probability Theory held in Berkeley 1970, Z. Frolík presented a lecture in which he gave very general conditions for this limit to exist in the class of all measure-spaces (σ-additive, of course).

From this brief survey it appears that an important feature of Frolík's papers is the applicability of his results both in topology and in other fields of mathematics. This is due to his wide interests which is also testified by his teaching activity at the Faculty of Mathematics and Physics of the Charles University. He leads seminars in complex analysis, in general and categorial topology, and also in abstract analysis the contents of which are probability spaces, stochastic processes, potential theory etc.

In conclusion, I would like to congratulate Z. Frolík, on behalf of mathematical community, upon the award of the State Prize and wish him to attain many other outstanding results.

Miroslav Hušek, Praha

SIXTIETH BIRTHDAY OF PROFESSOR ALOIS URBAN

Professor RNDr. Alois Urban, Head of Department of Descriptive Geometry of the Faculty of Mechanical Engineering of the Czech Technical University in Prague, reached sixty years of age on October 9, 1972.

Prof. Urban's field of scientific interest includes especially differential geometry, folowing the work of E. Čech, J. A. Schouten and V. Hlavatý, and descriptive geometry. Besides his scientific work, he is deeply involved in his educational activities being author or co-author of a number of lecture notes and textbooks for both universities and high schools, as well as in his work in the Union of Czechoslovak Mathematicians and Physicists. He is member of the Editorial Board of the journal Časopis pro pěstování matematiky (Journal for Cultivation of Mathematics).

A more detailed analysis of Prof. Urban's scientific and pedagogical activity may be found in the article *Šedesát let prof. Aloise Urbana* by Z. Vančura, Čas. pěst. mat. *97* (1972), 438—443.

Editorial Board

THIRD PRAGUE TOPOLOGICAL SYMPOSIUM 1971

From August 30 to September 3, 1971, the Czechoslovak Academy of Sciences organized in Prague the Third Symposium on General Topology and its Relations to Modern Analysis and Algebra. This Symposium followed two Topological Symposia held in Prague in 1961 and 1966 and was thematically allied to them. The Prague Topological Symposia have already become well known among topologists for bringing together prominent topologists as well as younger mathematicians many of whom have recently become known for their brilliant results. 107 mathematicians from 18 foreign countries and 51 Czechoslovak mathematicians attended the Third Prague Topological Symposium. There were 41 participants from the United Kingdom, the United States and Canada. Several participants were guests of the Czechoslovak Academy of Sciences, the Slovak Academy of Sciences, the Faculty of Mathematics and Physics of the Charles University, and the Association of Czechoslovak Mathematicians and Physicists. The International Mathematical Union granted a contribution towards travel expenses of invited speakers.

The program of the Third Prague Topological Symposium included one hour lectures and twenty minutes communications covering the latest trends in Topology. One of these trends is the theory of shapes. Another new trend that aroused attention and interest of mathematicians is infinite-dimensional topology. The third is the theory of compact spaces which has brought the solution to very difficult problems of general topology and a number of surprising results. Among other fields that were given prominence at the Symposium was the generalization of metric spaces and topological methods used in measure theory. There was a series of lectures on applications of topology.

At the Symposium, 22 invited lectures have been presented. The names of the lecturers and the titles of their lectures are listed as follows:

Anderson, R. D.: Some open questions in infinite-dimensional topology

Antonovskij, M. Ja.: Несимметрические близости, равномерности и разрывные метрики

Arhangelskii, A. V.: On cardinal invariants

Arya, S. P.: Sum theorems for topological spaces

Banaschewski, B.: On profinite universal algebras

Borsuk, K.: Some remarks concerning the theory of shape in arbitrary metrizable spaces

Frolík, Z.: Topological methods in measure theory and the theory of measurable spaces

de Groot, J.: On the topological characterization of manifolds

Herrlich, H.: A generalization of perfect maps

Hewitt, E.: Harmonic analysis and topology

Jones, F. B.: The utility of empty inverse limits

Katětov, M.: On descriptive classification of functions

Kuratowski, K.: A general approach to the theory of set-valued mappings

Mardešić, S.: A survey of the shape theory of compacta

Michael, E.: On two theorems of V. V. Filippov

Nagata, J.: A survey of the theory of generalized metric spaces

Pietsch, A.: Ideals of operators on Banach spaces and nuclear locally convex spaces

Pták, V.: Banach algebras with involution

Steiner, A. K.: On the lattice of topologies

Taylor, J. C.: The Martin compactification in axiomatic potential theory

West, J. E.: Identifying Hilbert cubes: General methods and their application to hyperspaces by Schori and West

Zarelua, A. V.: On infinite-dimensional spaces

During the Symposium, a reception was given at the Ministry of Education for a number of foreign and Czechoslovak participants.

The Symposium was closed by a speech of K. Kuratowski at a plenary session on September 3. In the evening, a banquet was given for all participants by the Chairman of the Czechoslovak Academy of Sciences, J. Kožešník. The following day was devoted to a coach trip to the North-East of Bohemia.

Most of the communications presented at the Symposium as well as a few submitted in writing only will appear in the Proceedings of the Third Prague Topological Symposium.

Josef Novák, Praha